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ROYAL ASTRONOMICAL SOCIETY

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MEETING OF 1934 NOVEMBER 9

Prof. F. J. M. Stratton, President, in the Chair.

The election by the Council of the following Fellows was duly confirmed :—

Ernest Valentine Payne, 17 Carson Road, Dulwich, London, S.E. 21; and Lieut. E. H. Thompson, R.E., 59th (Field) Coy., R.E., Canterbury.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Rev. Robert B. Bousfield, M.A., Ardath, College Road, Stanthorpe, Queensland, Australia (proposed by F. J. M. Stratton) ;

Anil Kumar Das, M.Sc., D.Sc., Solar Physics Observatory, Cambridge, and Meteorological Office, Poona, India (proposed by F. J. M. Stratton) ;

Murray Geddes, Otekura, Clutha County, Otago, New Zealand (proposed by R. A. McIntosh) ;

Robert Edward Diggles, B.Sc., The High School, Hanley, Stoke-on-Trent (proposed by R. Barker) ;

Rev. James E. Harrison, The Manse, Smeaton, Victoria, Australia (proposed by J. Brittain) ;

Willy Hartner, Ph.D., Bad Homburg v. d. Hessen, Ferdinandstrasse 42, Germany (proposed by L. J. Comrie) ;

Frank Maurice Holborn, Cumberland Lodge, 1 Streatham Hill, London, S.W. 2 (proposed by W. H. Steavenson) ;

Alan Hunter, B.Sc., D.I.C., 213 Strone Road, Manor Park, London, E. 12 (proposed by A. Fowler) ;

Henry Ernest Kersey, 10 St. Nicholas Street, Ipswich (proposed by W. Goodacre) ;

S. Krishnamurti, B.A., 61 Selbourne Gardens, London, N.W. 4 (proposed by C. C. L. Gregory) ;

Rev. Daniel J. K. O'Connell, S.J., M.Sc., Riverview College Observatory, Sydney, N.S.W., Australia (proposed by W. F. Gale) ;

Sir Herbert J. Read, K.C.M.G., 11 Stafford Mansions, Buckingham Gate, London, S.W. 1 (proposed by F. J. M. Stratton) ;

V. M. Sundararajan, M.Sc., 40 Boyne Avenue, London, N.W. 4 (proposed by C. C. L. Gregory) ;

Frank Twyman, F.Inst.P., F.R.S., Stanhope, Eastbury Avenue, Northwood, Middlesex (proposed by F. W. Dyson) ;

Frank Lorimer Westwater, B.A., Emmanuel College, Cambridge (proposed by W. M. Smart) ; and

William Herbert White, M.A., B.Sc., A.R.C.S., 45 Bushey Park Road, Teddington (proposed by C. P. Butler).

Two hundred and thirty-eight presents were announced as having been received since the last meeting, including :—

John Bentley [1805], On the Hindu Systems of Astronomy, and their Connection with History in Ancient and Modern Times (presented by Dr. Lockyer) ;

Sir James Jeans, Through Space and Time ;

H. Spencer Jones, General Astronomy : 2nd edition ;

Sir Isaac Newton's Mathematical Principles . . . (Revision of Motte's translation, with historical and explanatory Appendix, by F. Cajori) ;

Radcliffe Observatory, Oxford, The Radcliffe Catalogue of Proper Motions in the Selected Areas 1 to 115.

THE DENSITY OF INTERSTELLAR CALCIUM AND SODIUM.

Sir A. S. Eddington, F.R.S.

1. The researches of Struve and of Plaskett and Pearce have shown that the intensity of the interstellar *K* line is correlated to the distance of the star in which it is observed. The intensity is found to increase with distance, as had been expected on the hypothesis that the line is produced by absorption in a more or less uniform cosmic cloud. At first sight it would seem a comparatively easy matter to calculate from the observed intensity of the line the density in space of the Ca^+ atoms which produce the absorption. Such calculations have been given by Struve * and by Struve and Gerasimovič.† But I find that owing to the combination of two effects, here called *central absorption* and *wing absorption*, the relation of intensity

* *M.N.*, 89, 567, 1929. † *Ap. J.*, 69, 7, 1929.

to density is not so simple as is usually supposed. In particular, between densities 10^{-31} and $5{\cdot}10^{-29}$ grm. per cu. cm. the intensity of the line is practically independent of the density. This result seems likely to be of importance, because the purely theoretical determination of the density and state of ionisation of the cloud leads us to expect that the density of Ca^+ will be near the upper limit of this range and the density of neutral *Na* will be near the lower limit.

I think that several points emerge in the theory which may help to guide the interpretation of observational results. The most definite suggestion is that the existence of a range where a 500-fold increase of density produces no sensible increase of intensity explains why the *D* line is nearly as strong as the *K* line—a phenomenon which has hitherto been regarded as an outstanding difficulty.

In this paper the term "intensity" refers to the total amount of light absorbed by the line, *i.e.* combined width and depth. Since the true line-contour is rarely, if ever, observed, this integrated absorption is the most usual observational datum.

Through the work of Plaskett and Pearce the practical investigation of the intensities of the interstellar lines has become closely associated with the study of galactic rotation. It is therefore appropriate that the theory should be treated from the same point of view. In the main investigation, §§ 2–8, it is supposed that the cloud has no other motion than that due to galactic rotation. The correction arising from the temperature velocities of the atoms is obtained and discussed in § 6 ; it affects chiefly the nearer stars, and stars in galactic longitudes where the rotation term vanishes. Irregular currents in the cloud are considered in § 9. Since the observational evidence indicates that there is a considerable amount of irregular motion, the results obtained for the simple model in §§ 2–8 are only expected to hold statistically.

2. In an absorption line centred at frequency ν_0, the mass-coefficient of absorption s_ν for light of frequency ν has the form *

$$s_\nu = \frac{s_0}{1 + a^2(\nu - \nu_0)^2}. \tag{1}$$

Consider light coming from a star at a distance l. Let ρ be the density (supposed uniform) of the atoms producing the absorption (1). Owing to the Doppler effect of the differential velocity due to galactic rotation, ν_0 will change uniformly along the track. Let $dl = A d\nu_0$, so that A is a constant ; and let ν_1, $\nu_2 (\nu_2 > \nu_1)$ be the values of ν_0 at the two ends of the track. Then

$$l = A(\nu_2 - \nu_1). \tag{2}$$

The optical depth between us and the star in frequency ν is

$$\tau_\nu = \int \rho s_\nu dl = A\rho \int_{\nu_1}^{\nu_2} s_\nu d\nu_0$$

$$= \frac{A\rho s_0}{a}\{\tan^{-1} a(\nu_2 - \nu) - \tan^{-1} a(\nu_1 - \nu)\} \tag{3}$$

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* See R. v. d. R. Woolley, *M.N.*, 90, 171. Woolley gives the formulæ and numerical values for s_0 and a.

by (1). For the K line, $a = 8{\cdot}68 \,.\, 10^{-8}$. Thus, for example, $a\Delta\nu = 1$ when the corresponding $\Delta\lambda$ is 0·00006 angstroms. For all but a minute part of the line $a(\nu_2 - \nu)$ and $a(\nu_1 - \nu)$ are numerically large, so that the corresponding angles in (3) are nearly $\pm \frac{1}{2}\pi$; and we have as a first approximation

$$\left.\begin{aligned} \tau_\nu &= \pi A \rho s_0 / a \quad \text{if} \quad \nu_2 > \nu > \nu_1, \\ &= 0 \qquad\quad \text{if} \quad \nu > \nu_2 \quad \text{or} \quad \nu < \nu_1. \end{aligned}\right\} \tag{4}$$

To examine the error, we proceed to a second approximation by writing

$$\tan^{-1} a(\nu_2 - \nu) = \frac{\pi}{2} - \frac{1}{a(\nu_2 - \nu)} \quad \text{or} \quad -\frac{\pi}{2} - \frac{1}{a(\nu_2 - \nu)},$$

according as the argument is positive or negative. The error of the coefficient (π or 0) is therefore

$$\frac{1}{a(\nu_1 - \nu)} - \frac{1}{a(\nu_2 - \nu)} = \frac{\nu_2 - \nu_1}{a(\nu_1 - \nu)(\nu_2 - \nu)}. \tag{5}$$

Near the ν_2 end of the line this becomes, to the first order in $\delta\nu$,

$$1/a\delta\nu,$$

where $\delta\nu = \nu - \nu_2$. So that the error of (4) is

$$\delta\tau = \tau_0 / \pi a \delta\nu, \tag{6}$$

where we have written

$$\tau_0 = \pi A \rho s_0 / a. \tag{7}$$

We shall call the intensity of the line arising from τ_0 *central* absorption, and the intensity arising from $\delta\tau$ *wing* absorption. By (4) the central absorption is limited to frequencies between ν_1 and ν_2. We shall see that it is not until long after the density has become sufficient to blacken out this part of the line that there is any appreciable prolongation of the line by wing absorption. We therefore first consider central absorption only.

3. *Central Absorption.*—If $I_\nu d\nu$ is the intensity of the star's continuous spectrum, supposed uniform over the width of the line, the total amount of the light absorbed is

$$\int_0^\infty (1 - e^{-\tau_\nu}) I_\nu d\nu = I_\nu (\nu_2 - \nu_1)(1 - e^{-\tau_0}) \tag{8}$$

by (4) and (7). If τ_0 is small, this reduces to

$$\begin{aligned} I_\nu (\nu_2 - \nu_1)\tau_0 &= I_\nu (\nu_2 - \nu_1)\pi A \rho s_0 / a \\ &= I_\nu \pi l \rho s_0 / a. \end{aligned} \tag{9}$$

In this case the amount absorbed is proportional to the density ρ of the absorbing atoms and to the distance l of the star. But if $\tau_0 > 2$ an increase of density scarcely affects the absorption, which is already 86 per cent. saturated. For this and higher density we can neglect $e^{-\tau_0}$, so that (8) becomes

$$I_\nu (\nu_2 - \nu_1) = I_\nu l / A. \tag{10}$$

I shall call the value of ρ which corresponds to $\tau_0 = 2$ the *saturation density*.

The theoretical (physical) value of s_0/a is *

$$\frac{s_0}{a} = \frac{e^2}{cm} \cdot \frac{f}{M}, \tag{11}$$

where M is the mass of the absorbing atom and f is the oscillator strength of the line. For the K line $s_0/a = 8{\cdot}54 \cdot 10^{19}$. Taking the galactic rotation effect to be 17 km. per sec. per kiloparsec, we find by (2)

$$A = 7{\cdot}1 \cdot 10^{10} \text{ cm. sec.} \tag{12}$$

Hence by (7) the saturation density is

$$\left.\begin{aligned} \rho = 2a/\pi A s_0 &= 1{\cdot}05 \cdot 10^{-31} \text{ for } Ca^+ \ (K \text{ line}) \\ &= 6 \cdot 10^{-32} \text{ for } Na \ (D \text{ line}). \end{aligned}\right\} \tag{13}$$

The latter value is for the stronger component of the D line (homologous with the K line), so that by (11) the saturation densities are simply proportional to the atomic weights M of *Ca* and *Na*.

Since A depends on galactic longitude, these saturation densities refer to galactic longitudes in which the rotation effect is a maximum. In other longitudes the saturation density is smaller. For statistical discussions the average saturation densities should be taken to be $2/\pi$ times the above values, viz.

$$Ca^+, \ 6{\cdot}7 \cdot 10^{-32}; \qquad Na, \ 3{\cdot}8 \cdot 10^{-32}. \tag{14}$$

In practice, however, the lines are prolonged by temperature motions of the atoms and by irregular motions in the cloud, and therefore require rather more density to saturate them. These additions would roughly compensate the factor $2/\pi$, so that we shall generally regard the values in (13) as the typical saturation densities.

4. The very low saturation density makes it probable that the K line is fully saturated and that the D line is either saturated or nearly so. This is illustrated by the results obtained in my theoretical study of the cosmic cloud.† I found 10^{-24} for the whole density of the cloud; also that 1 calcium atom in 830 would be singly ionised. Taking the abundance of calcium to be 1·5 per cent. (as given by C. H. Payne), the density of Ca^+ becomes $2 \cdot 10^{-29}$, so that there is saturation with a 200-fold margin. The corresponding abundance of sodium is 2 per cent., and 1 atom in 300,000 is neutral. This gives a density $7 \cdot 10^{-32}$ for *Na*, so that there is just saturation. Neutral *Ca* has 1/67000 of the density of Ca^+, or $3 \cdot 10^{-34}$; the neutral line should therefore have 1/170 of the intensity of the ionised line.

Results which involve estimates of abundance are necessarily rough. Apart from this uncertainty, the figures are fairly definite, and would not, I think, be very much altered by a modern recalculation. I had thought that the recent recognition of the great abundance of hydrogen would introduce considerable changes in this as in most branches of astrophysics. But I

* Woolley, *loc. cit.*, equation (2.42).

† Bakerian Lecture, *Proc. Roy. Soc.*, A, **111**, 424, 1926. The figures quoted here are from a somewhat later calculation in *Der Innere Aufbau der Sterne*, p. 479, 1928.

find that in a normal region of interstellar space the hydrogen will be entirely un-ionised, and indeed in molecular form. The intensity of the ionising radiation will remain as assumed in my earlier calculations, except that photons of energy greater than 13·5 volts will be entirely absent, having been absorbed and transmuted by the hydrogen near the stars. Their absence has no important effect on our results.

The observational evidence imposes a rather strict censorship on any suggested alterations of the above figures. Substantial changes in one direction would enfeeble the sodium line too much ; substantial changes in the other direction would make the neutral calcium line occasionally visible.

The most important result is that there is no longer any difficulty in accounting for the approximately equal intensities of sodium and calcium lines. The difficulty had been that the conditions of ionisation in the cloud are almost certainly such as to make the neutral sodium atoms (absorbing the *D* line) much less numerous than the singly ionised calcium atoms (absorbing the *K* line). As nearly as we can calculate, the ratio is 1 : 300. According to Struve the assumed abundance of calcium (1·5 per cent.) was about the right amount to account for the intensity of the *K* line ; hence we cannot equalise the densities even by arbitrarily postulating greater abundance of sodium—unless we postulate an abundance greater than 100 per cent.! We now see that the equal intensity of the lines does not imply equal density. The anticipated density of Na is just about sufficient to give a saturated line, and the 300-fold greater density of Ca^+ cannot produce greater strength.

According to C. S. Beals * the sodium lines are actually stronger than the calcium lines. But his intensities are measured in units of wave-length, and must be divided by λ in order to afford a fair comparison. When so transformed, the average intensities of sodium and calcium are about the same.

5. We have now to examine whether the observed intensity of the *K* line agrees in absolute amount with that given by our formula (10). We can save ourselves a laborious examination of the observational data by adapting Struve and Gerasimovič's discussion. They deduced a density of Ca^+ from the observed intensities. If the line is produced by central absorption their deduced density is spurious, but it is none the less a convenient intermediary for comparing our theory with the observed intensities used in their discussion.

Struve and Gerasimovič found for Ca^+ a density $\rho = 3{\cdot}6 \,.\, 10^{-32}$. This result is easily explained on the present theory. If we calculate the amount of absorption without regard to saturation, *i.e.* by using a linear formula $1 - \tau$ instead of an exponential formula $e^{-\tau}$, a perfectly black line corresponds to $\tau = 1$. Conversely, if we have a saturated line whose total intensity is observed, and (not realising that it is saturated) we compute the number of atoms responsible for it by the linear absorption formula, we shall obtain that density of the atoms which would make $\tau = 1$ at all parts of the line.

* *M.N.*, 94, 663, 1934.

The density of Ca^+ which would give $\tau = 1$ is $3{\cdot}3 \cdot 10^{-32}$, *i.e.* half the saturation density ($\tau = 2$) given in (14). This nearly represents the procedure of Struve and Gerasimovič. They found a coefficient of absorption $1{\cdot}1 \cdot 10^{-22}$ per cm., so that optical depth unity would correspond to 3000 parsecs. Assuming that their material refers in the mean to a distance of 1500 parsecs, their corresponding absorption factor is $(1 - e^{-\frac{1}{2}})$ or 0·4, whereas the factor corresponding to the linear absorption formula is 0·5. Thus the density computed from the observed intensities by a linear formula is $\frac{4}{5}$ of Struve and Gerasimovič's value, viz. $2{\cdot}9 \cdot 10^{-32}$.

We find therefore that (when both are treated by the same linear absorption formula) the theoretical intensities give $\rho = 3{\cdot}3 \cdot 10^{-32}$ and the observed intensities employed by Struve and Gerasimovič give $\rho = 2{\cdot}9 \cdot 10^{-32}$. Regarded as determinations of the density of Ca^+ these values of ρ are spurious ; but that does not affect their usefulness in showing that there is close agreement in the absolute value of the theoretical and observed intensities to which they respectively correspond. If we apply a correction, discussed in § 6, for velocity spread due to temperature, the agreement is not quite so close ; but it is probably as close as the accuracy of the observational data warrants.

We may also examine the measurements made by E. G. Williams,* which refer to comparatively near stars. It is often convenient to express line-intensity in km. per sec. ; *i.e.* we take the width of a perfectly black line which would give the total absorption measured, and state the velocity which would cause a Doppler displacement equal to this width. According to (10) the width of the line should be equal to the differential velocity of galactic rotation for the star. Including the averaging factor $2/\pi$, this would be 11 km. per sec. per 1000 parsecs. Williams's unit is very nearly 1 km. per sec. His intensities are mostly between 0 and 25 units. He used bright stars which must be comparatively near—perhaps within 500 parsecs, but it is difficult to estimate. For these near stars the intensity of the lines is sensibly increased by temperature prolongation (§ 6). On the whole we may conclude from Williams's measurements that the observed intensities are somewhat greater than the theoretical intensities, but probably not more than three times as great.†

6. *Temperature Prolongation.*—In my earlier work I have estimated the molecular temperature of the cosmic cloud at 15,000°. This figure is particularly difficult to determine, and it might possibly be much lower, say 4000°. Such a change would have no great effect on the ionisation conditions, but it would halve the temperature prolongation here determined.

At 15,000° the radial velocities of the calcium atoms follow the distribution law $e^{-h^2v^2}$ where $1/h = 2{\cdot}5$ km. per sec. Since we suppose that the density of Ca^+ is at least 100 times the saturation density, there will be sufficient high-speed atoms to prolong the line appreciably beyond the limits ν_1 and ν_2. We may take the line to end at the points where saturation ceases, for the number of atoms capable of absorbing beyond these points

* *Ap. J.*, 79, 280, 1934, Table VII.

† Four stars (out of 67) show exceptionally great intensity.

falls off very rapidly. We easily find that for density 100 times the saturation value, the prolongation of the line is 4 km. per sec. at each end ; for 1000 times it is $5\frac{1}{2}$ km. per sec. ; and for 10,000 times it is $6\frac{1}{2}$ km. per sec.

Approximately therefore the effect of temperature is to give the K line a minimum intensity corresponding to a width of 8 to 10 km. per sec. This is added on to the intensity found in the foregoing discussion. It will appreciably increase the intensities for the nearer stars, and have relatively less effect as the distance increases. It cannot be strictly independent of distance, *e.g.* it would not apply to a star at 1 parsec distance ; but from 100 parsecs upwards it does not change much.

Since temperature prolongation depends on the line being heavily supersaturated, there is no corresponding effect on the D line. The D lines should therefore be relatively less intense than the K lines in the nearest stars.

7. *Wing Absorption.*—The additional absorption given by (6) can prolong a saturated line beyond the limits ν_1 and ν_2. This wing prolongation does not terminate so abruptly as temperature prolongation. For a rough computation I shall take the termination to be at $\tau = 1$, so as to include a certain amount of unsaturated absorption, setting off the defect of saturation against the faint prolongation beyond $\tau = 1$. Then by (6) the prolongation is

$$\delta\nu = \tau_0/\pi a. \tag{15}$$

If $\delta\nu$ represents 1 km. per sec., we find $\tau_0 = 690$; this corresponds to nearly 350 times the saturation density. Thus wing absorption should not begin to be appreciable until the density is considerably higher than the densities that we have been contemplating. (We shall find, however, that this method of calculation rather underestimates the effect of wing absorption.)

We next calculate the line intensity on the hypothesis that wing absorption alone is responsible for it ; that is to say, we neglect all motions of the atoms.* Then ν_0 is constant in (1), so that

$$\tau_\nu = \rho l s_\nu = \rho l s_0/a^2(\nu - \nu_0)^2. \tag{16}$$

The approximation $s_\nu = s_0/a^2(\nu - \nu_0)^2$ is here sufficient ; it only fails near the centre of the line—a region which is blackened out in any case. The total absorption is

$$\int I_\nu(1 - e^{-\tau_\nu})d\nu = I_\nu(\rho l s_0/a^2)^{\frac{1}{2}}\int_{-\infty}^{\infty}(1 - e^{-1/x^2})dx$$

$$= I_\nu \,.\, 2\sqrt{(\pi\rho l s_0/a^2)} \tag{17}$$

since the definite integral is $2\sqrt{\pi}$.

By comparing (10) and (17) we can find the density which is such that central absorption and wing absorption give the same value of the line intensity, viz.

$$\rho = \frac{a^2(\nu_2 - \nu_1)^2}{4\pi l s_0} = \frac{a^2 l}{4\pi s_0 A^2}. \tag{18}$$

Taking a galactic longitude where the rotation effect is a maximum, we have for $l = 1000$ parsecs

* This calculation is the same as that of Struve, *M.N.*, **89**, 584, 1929.

$$\rho = 4{\cdot}9 \cdot 10^{-29}.$$

We note that ρ is proportional to l.

The central absorption and wing absorption are not additive ; but when they are nearly equal the intensity must be a little greater than that given by either of them acting alone. It follows that at density $5 \cdot 10^{-29}$ the intensity of the lines in stars within 1000 parsecs distance will have just begun to increase above the steady value previously discussed. At higher density wing absorption predominates ; and ultimately the intensity increases as the square root of the density by (17).

As illustrating the distribution of intensity in wing absorption, we find from (16) that when $\rho = 5 \cdot 10^{-29}$, $l = 1000$ parsecs, the saturated part of the line has a width 7 km. per sec. ; so that the total intensity (17 km. per sec.) includes 10 km. per sec. contributed by unsaturated extensions. The magnitude of the unsaturated extension is greater than was allowed for when we supposed the line to terminate at $\tau = 1$ at the beginning of this section, so that the result there found was an underestimate.

8. *Relative Intensities of* H *and* K *Lines.*—The whole course of the change of intensity of the *K* line with the density of Ca^+ can now be traced. The numerical values given below refer to galactic longitudes where the rotation effect is a maximum :— *

Range A (unsaturated central absorption). At or below $\rho = 2 \cdot 10^{-32}$ the intensity increases proportionately to the density. The rate of increase falls off as ρ approaches 10^{-31}.

Range B (saturated central absorption). Between $\rho = 10^{-31}$ and $\rho = 5 \cdot 10^{-29}$ the intensity is practically constant. (The upper limit applies to a distance of 1000 parsecs and is proportionately lower for smaller distances.)

Range C (wing absorption). Above $\rho = 5 \cdot 10^{-29}$ the intensity begins to increase again, and ultimately increases proportionately to the square root of the density.

Our rough theoretical estimate of the actual density of Ca^+ indicates a point in range B rather near the upper limit. But it has been pointed out to me by E. G. Williams that there is very definite evidence that the *H* line is less intense than the *K* line, whereas they should have the same intensity if they both correspond to saturated central absorption.

This evidence seems decisive that the density cannot actually fall in range B, at any rate for the nearer stars examined by Williams. As regards the choice between A and C, we may note that in range A the intensity-ratio K/H can reach 2, whereas in range C it cannot exceed $\sqrt{2}$. Although observers would probably prefer a value greater than $\sqrt{2}$, I think that the absolute intensities of the lines clearly indicate range C. We have seen that Williams's measures give an intensity greater than the theoretical intensity for range B.

* Ranges A and C have been studied by Unsöld, Struve and Elvey, *Zeits. für Astrophysik*, **1**, 314, 1930 ; but range B seems to have escaped notice hitherto.

The density cannot be very much greater than $5 \cdot 10^{-29}$. For example, a density $1{\cdot}5 \cdot 10^{-28}$ gives by (17) an intensity of 17 km. per sec. at 330 parsecs. This seems definitely greater than the observed intensities. I suggest that $8 \cdot 10^{-29}$ would fit most of the conditions. It implies, however, that the difference of intensity of K and H is only prominent in the nearer stars, and that the two lines approach equality at distances beyond 1000 parsecs.

If the two D lines are of unequal intensity the weaker line must be unsaturated. The weaker line would become saturated at density $1{\cdot}2 \cdot 10^{-31}$, so that the density of neutral sodium must be below this value.

If the cloud is uniform, we may anticipate that the intensity ratio K/H will diminish as the distance increases, but the intensity ratio D_2/D_1 will be independent of distance.

There may also be a variation with galactic longitude which should be opposite for K/H and for D_2/D_1. In longitude 330° central absorption for Ca^+ gives only a narrow line; thus wing absorption strongly predominates, and the intensity-ratio K/H tends to its maximum value $\sqrt{2}$. In the same longitude the saturation density is a minimum, so that both the D lines tend to be fully saturated, and the intensity ratio D_2/D_1 tends to 1.

9. *Radial Velocities.*—From the measured radial velocities of the K lines it appears that there are considerable irregular motions in the cloud in addition to galactic rotation. This makes it difficult to forecast the behaviour of individual stars. There is, however, one condition which ought to be fulfilled by the stars individually almost without exception.

If the intensity of the K *line is measured in velocity units, the radial velocity cannot exceed half the intensity.*

We refer here to the residual radial velocity corrected for solar motion. The measurement of intensity in km. per sec. has been explained in § 5.

Consider first central absorption. The K line should in all cases be a continuous black line extending between positions corresponding to the greatest and least velocities of the material traversed by the light. A gap could only occur if there were at some point of the track a fast-moving current introducing a velocity gradient greater than 1 km. per sec. per parsec. For we have seen that the velocity gradient due to galactic rotation, viz. 1·7 km. per sec. per 100 parsecs, gives a continuous saturated line when the density of the Ca^+ is 10^{-31}; and since the actual density is at least 200 times greater, it suffices for a 200 times greater velocity gradient. It seems very unlikely that any current in the cloud could be so sharply bounded. Moreover, unless the total discontinuity exceeded 8 km. per sec., the gap would be filled up by temperature prolongation.

The line must always include the undisplaced position which corresponds to absorption in our immediate neighbourhood. If it extends on one side only of this position, the observed radial velocity (or displacement of its centre of gravity) is evidently half its total width. If it extends on both sides, the radial velocity is less than half the total width.

Wing absorption will scarcely affect this relation. It extends the saturated part of the line without leaving any gaps. Theoretically an

unsaturated wing extension at the high-velocity end of the line might give a displacement of the centre of gravity proportionately greater than the addition to the total intensity. But it is doubtful if it would have the same effect on practical measurements of the displacement; and in most cases there would be a compensating effect at the other end of the line. In fact, if a line has unsymmetrical unsaturated wings its radial velocity becomes a rather indeterminate conception.

The result which we have enunciated should therefore hold in all cases. It results from the fact that, in a uniform line which includes the origin, the distance of the centre of gravity from the origin cannot be more than half the width of the line.

10. *Galactic Longitude Effect.*—In current investigations the correlation between intensity and radial velocity has been chiefly studied through an intermediate investigation of the galactic rotation. But, as illustrated in § 9, there is a more direct correlation.

In range B (saturated central absorption) there is no direct relation between intensity and distance. The direct relation is between intensity and radial velocity; this gives an indirect relation between intensity and distance because, owing to galactic rotation, large distances give high average radial velocity.

In these circumstances there should be a strong longitude effect. Considering stars at the same distance, the intensity of the *K* line should vary with galactic longitude, being a maximum in longitudes 15°, 105°, etc., where the radial velocities are large, and a minimum in longitudes 330°, 60°, etc.

We do not expect to find this effect in the brighter stars because we have found it necessary to place them in range C (in order to account for the intensity ratio *K*/*H*, and for the slight excess of the observed intensity over the theoretical value for saturated central absorption). But at greater distances central absorption increases faster than wing absorption, and we should estimate that at 1500 parsecs the conditions correspond to range B. It might therefore be worth while to search for a galactic longitude effect in the more distant stars.

11. *Conclusion.*—We have examined the way in which the intensity of an interstellar line depends on the density of the absorbing atoms, and have found that there is 500-fold range extending from 10^{-31} to about $5 \cdot 10^{-29}$ grm. per cu. cm. over which the intensity remains practically constant. It appears that the densities of ionised calcium and neutral sodium fall near the upper and lower ends of this range. This accounts for the approximate equality of intensity of the *K* and *D* lines notwithstanding a great difference of density. The upper limit of the range of constant intensity varies in proportion to the distance of the stars considered. It seems that Ca^+ falls a little above the limit for the near stars, but below the limit for stars beyond 1000 or 1500 parsecs.

Several observational tests are suggested which, if they could be carried out with sufficient accuracy, would fix the position of the *K*, *H* and *D* lines in relation to this range, and so give a very close determination of the density in space of Ca^+ and *Na*.

ON THE CONSTITUTION AND DEVELOPMENT OF ROTATING STELLAR SYSTEMS.

Bertil Lindblad.

Modern researches on the structure of the stellar system have strongly emphasized the enormous flattening of the system towards the galactic plane. The dispersion in distance from this plane of a couple of hundred parsecs at our point of the system is quite insignificant compared with the dimensions of the system in the plane of the Milky Way, which we estimate to be at least 30,000 parsecs. A similar phenomenon is apparent in the case of many extra-galactic nebulæ, *e.g.* our neighbouring system, the great nebula in Andromeda. Here Stebbins and Whitford's * recent measurements have greatly increased the apparent dimensions of the object, and it seems plausible that this extension corresponds in reality to a continuation of the thin disk formed by the earlier known outer regions of the system. In two recent papers † the author has advanced the opinion that the exceedingly flat disks of the systems in question may be considered as a remnant of a wider distribution which has become exceedingly flattened towards the fundamental plane of rotation of the system by a process which we may name a "sedimentation" of matter towards the plane of rotation. This theory forms, moreover, a natural starting-point for a discussion of spiral structure as a consequence of a partial instability of the system, in general conformance with a hypothesis I had proposed earlier. It seems that at some important points of the discussion the arguments may be drawn considerably closer, and I proceed to discuss these points here, connecting up to the general analysis of the previous paper. The method used is thus the same, viz. a generalization of the method advanced by the writer in developing the principles of the rotation theory of the Galaxy.

We introduce a co-ordinate system x, y, z, with the centre of gravity of the stellar system as origin and the invariable plane of the system as xy plane. We further pass to cylindrical co-ordinates R, θ, z, by putting

$$x = R\cos\theta, \qquad y = R\sin\theta.$$

The linear velocities in the R, θ and z directions are denoted by Π, Θ, Z. The potential function $V(x, y, z, t)$ is determined at each moment, t, by Poisson's equation, $\nabla^2 V = -4\pi G\rho$, where ρ is the density of matter at x, y, z. Forming the quantities

$$I_1 = \Pi^2 + \Theta^2 + Z^2 - 2V, \qquad I_2 = R\Theta, \tag{1}$$

and also the total kinetic energy T and the potential energy W, as follows, m being the mass of an individual particle,

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* *Proc. Nat. Ac. Sc.*, **20**, 93, 1934.

† *Arkiv f. Matematik, Astronomi och Fysik*, **23A**, No. 18, 1933 (*Stockholm Medd.*, No. 11); *M.N.*, **94**, 231, 1934.

$$T = \tfrac{1}{2}\Sigma m(\Pi^2 + \Theta^2 + Z^2), \qquad W = \tfrac{1}{2}\Sigma m V,$$

we have by the virial theorem in the mean over a sufficiently long time

$$\Sigma m I_1 = 6(T - W) = -6T = -3W = \text{const.} \tag{2}$$

We have further the total moment of momentum

$$\Sigma m I_2 = \text{const.} \tag{3}$$

We find easily that the general expressions of the variations ΔI_1 and ΔI_2 for an individual particle may be written

$$\Delta I_1 = -2\int_c \frac{\partial V}{\partial t} dt, \qquad \Delta I_2 = \int_c \frac{\partial V}{\partial \theta} dt, \tag{4}$$

where the integrals are performed along the orbit, c, of the particle.

We may further perceive as an empirical result, from our knowledge of those types of flattened stellar systems which we find in nature, that the potential V is likely to be determined to a very great extent by a huge central mass of the system. Under the dominating influence of the central mass $\frac{\partial V}{\partial t}$ and $\frac{\partial V}{\partial \theta}$ may be considered as small quantities, and we may accordingly expect for long times only small variations ΔI_1 and ΔI_2. If we consider further the vertical motion across the galactic plane, we can define

$$Z_0^2 = Z^2 - 2[V(R, z) - V(R, 0)], \tag{5}$$

and then identify Z_0 to be nearly identical with the vertical velocity component at the moment, when the star traverses the galactic plane $z = 0$. The quantity Z^2_0 may further be considered nearly constant for an individual particle which does not depart very far from the galactic plane.

Instead of the co-ordinates and velocity components

$$R, \quad \theta, \quad z, \quad \Pi, \quad \Theta, \quad Z$$

we now introduce as variables

$$R, \quad \theta, \quad z, \quad I_1, \quad I_2, \quad Z_0^2. \tag{6}$$

This substitution of variables is of importance because by using the variables (6) we can get a comprehensive view of the density of matter and of the velocity distribution at all points of the system simultaneously.

We achieve this by observing, in the way of previous papers,* that possible combinations of I_1 and I_2 for a given point of the system are confined to a region having as boundaries $I_1 = 0$ and the "characteristic parabola" $I_2^2 = R^2(I_1 + 2V)$. The successive parabolas for the points situated on a radius in the galactic plane generally have an envelope curve E. The tangent point between parabola and envelope corresponds to a "circular" motion, $\Pi = Z = 0$, $\Theta = \Theta_c$, where

$$\Theta_c^2 = -R\frac{\partial V}{\partial R}. \tag{7}$$

* Cf. *M.N.*, 94, fig. 1, p. 232, 1934.

We define here as "circular" motion the instantaneous motion in the Θ direction, for which the centrifugal force $\frac{1}{R}\Theta^2$ balances the component of force along the radius. The common tangent of parabola and envelope has an inclination ϕ with respect to the I_2 axis determined by the equation

$$\tan \phi = 2\omega_c, \tag{8}$$

where $\omega_c = \frac{\Theta_c}{R}$. As the potential V for given R is likely to be a maximum in the galactic plane, the envelope curves for various radii map out the entire field of the I_1, I_2 plane in which combinations of I_1 and I_2 may occur. We remark that Z_0^2 vanishes uniformly on the envelope. Further, it has a constant value on any characteristic parabola. We accordingly have $Z_0 = 0$ on any characteristic parabola touching the envelope.

The possible distribution in R for given Z_0^2 may be studied by forming

$$I_1' = I_1 - Z_0^2 = \Pi^2 + \Theta^2 - 2V(R, 0), \tag{9}$$

and then considering the characteristic diagram in I_1' and I_2 in the same way as in I_1 and I_2, forming the characteristic parabolas $I_2^2 = R^2(I_1' + 2V(R, 0))$ with their envelope.

As to the distribution in z we observe that we have the condition

$$Z_0^2 \geqslant 2[V(R, 0) - V(R, z)]. \tag{10}$$

We further remark that for a given R the possible distribution in z depends entirely on the distribution in Z_0^2 for the matter which lies within the characteristic parabola for $z = 0$, viz. $I_2^2 = R^2(I_1 + 2V(R, 0))$. This parabola, on which $Z_0 = 0$, includes within its domain all parabolas of the same R and of $|z| > 0$.

It may be observed that our development so far is perfectly general and does not depend on any assumption regarding a steady state of the system. We have now to consider, however, the incoherence of the motion of the various particles of the system, which leads to an ultimate disintegration of every chance accumulation of matter in the system. The generalized 3-dimensional surface

$$I_1 = \text{const.}, \qquad I_2 = \text{const.}, \qquad Z_0^2 = \text{const.} \tag{11}$$

in the 6-dimensional space formed by the co-ordinates x, y, z, X, Y, Z, is of finite size for $I_1 < 0$, and, as long as ΔI_1, ΔI_2, ΔZ_0^2 are small, we must expect the 6-dimensional orbits of the various particles to remain for a long time in the close neighbourhood of one and the same surface. Ordinarily we may assume that the orbit in question covers entirely the surface (11). On account of the incoherence of the motions we must consider as a matter of chance just where on the surface (11) the particles of a certain phase element $dxdydzdXdYdZ$ are situated at a certain moment.

We must therefore admit that the most probable frequency *a priori* within an element of volume $dxdydzdXdYdZ$ or $RdRd\theta dzd\Pi d\Theta dZ$ must

be considered as one and the same function of I_1, I_2 and Z_0^2, which we denote by

$$f(I_1, I_2, Z_0^2). \tag{12}$$

The amount of matter to be expected *a priori* per unit volume at a certain point R, θ, z will then be the total frequency which according to this distribution function falls within the characteristic parabola of the point in question.

It is well known that we get an explanation of the general distribution of stellar velocities at our point of the galactic system, as well as of the concentration of the stars towards the galactic plane, if we assume a general concentration of the frequency function f towards the envelope curve in the I_1, I_2 diagram.* This is, in fact, an expression of the fundamental assumption of the rotation theory of the Galaxy. A few important consequences of this theory may be summed up very briefly as follows.

A preferential motion relative to the centroid occurs in a direction along the radius vector from the centre of the system, and depends essentially on the curvature of the envelope E, which in its turn, according to (8), measures the variation of the angular velocity ω_c with the distance R from the centre. The same phenomenon, the curvature of E, causes the positive coefficient A of Oort's differential rotation, so that we have a definite relation between the two axes σ_1 and σ_2 of the velocity ellipsoid in the galactic plane and the coefficient A. We have, in fact, the relation

$$\frac{\sigma_2}{\sigma_1} = \sqrt{1 - \frac{A}{\omega}}, \tag{13}$$

where for small dispersions the mean angular velocity ω approaches to ω_c.

The asymmetrical drift of large velocities relative to the centroid is also well accounted for, and seems to depend ultimately on the influence of the velocity limit $I_1 = 0$.

We now proceed to discuss the hypothesis advanced in previous papers that the sedimentation towards the envelope in the I_1, I_2 diagram is to a considerable extent due to the reaction upon the system by an ultimate ejection of such particles which approach slowly, as a consequence of an accumulation of perturbations of their motions, to the velocity limit $I_1 = 0$. The particles will depart mainly from the outer regions of the system, and preferably near the very "edge" of the system. Therefore there will be a decrease of potential V and an increase of $\frac{\partial V}{\partial R}$ in the outer regions. This will lead to a contraction of the system (as can be predicted already from the virial theorem). In the I_1, I_2 diagram the envelope curve will rise in its upper course and sink in its lower part, so that the new envelope curve intersects the old one.† The simplest example of such a process appears to be the following one. Suppose we arrange a number of material particles in a plane formation of circular symmetry in such a way that the force of

* Cf. *e.g.* "Die Milchstrasse," *Handbuch der Astrophysik V* (2), 1933.

† Cf. *M.N.*, 94, fig. 2, p. 239, 1934.

gravitation along the radius R varies proportionally to R. We may then give to the entire assembly a suitable angular velocity of rotation ω_c such that the particles describe circular orbits around the centre. We may make the assembly sufficiently large, until we get $I_1 = 0$ for the motion of the outermost particles. The conditions are then exactly those of an infinitely flattened spheroidal distribution. The envelope in the I_1, I_2 diagram for such a system (fig. 1) will be a straight line with the angle of inclination $2\omega_c$. If we give to the particles moving at the edge of the system an infinitely slight disturbance, these particles will depart in spiral orbits towards infinity. The potential V in the outer regions decreases, the system contracts, and the

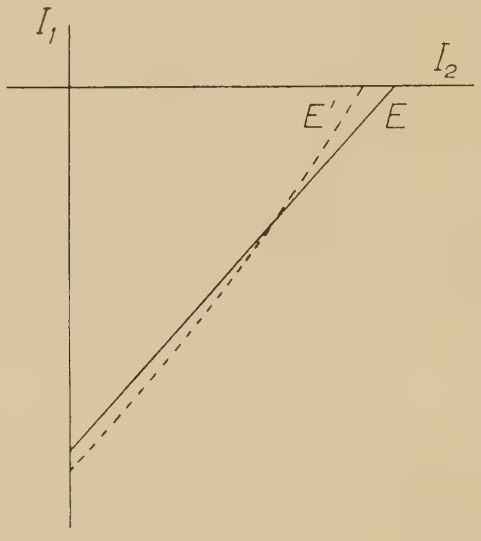


Fig. 1.

new envelope E' will proceed as indicated by the dotted line in the figure. If the process is slow, the circular motions of the particles will be preserved.

In the actual case of our stellar system there is of course a dispersion of the velocities relative to the circular motions. It may be expected, as indicated in one of the previous papers, that the change of the run of the envelope and the simultaneous regular change in I_1 according to (4) for a particle inside the envelope will on the average co-operate in such a way that this dispersion decreases in the outer regions and increases in the inner regions of the system.

This process, however, does not cover the whole point at issue. We have not only to consider the immediate effect of the ejection of particles which originally lie near $I_1 = 0$, but further also the way in which the ejected particles get supplied by particles emanating from the mass of stars of lower I_1. We have thus to follow a certain net stream of particles at different levels in I_1, and consider the recoil effect on the remaining matter of the disturbances which propagate this net stream. The net stream measures the surplus of particles passing a certain value of I_1 in the direction from lower to higher

values over those going in the opposite direction, and obviously arises from the deviation from a steady distribution which is caused by the deficiency of particles of large I_1. The net stream will of course decrease gradually with decreasing level I_1, and must accordingly be assumed to decrease slowly with time, as the mean value of I_1 decreases. If now the individual disturbances in I_1 and I_2 at a certain point in the diagram be ΔI_1 and ΔI_2, the corresponding recoil effects on the surrounding matter (actually working upon that part of the matter which we assume to form the disturbing condensations) are of course the same with opposite sign, thus $-\Delta I_1$ and $-\Delta I_2$.

Very much will now depend on the average "type" of the disturbances. By this we mean the ratio $\frac{\Delta I_1}{\Delta I_2}$ which evidently characterizes the disturbance as well as the recoil effect. To discuss this point we must inquire more closely into the nature of the perturbations. As a physical cause of the disturbances in question we do not, at least in the first place, consider passages between individual stars. Instead we take into account the disturbing effects due to star-clusters, star-clouds and nebulous or meteoric clouds on the orbit of a star. According to our ideas concerning the nature and general state of motion of the stellar strata in our system and in the extra-galactic nebulæ we infer as an unvoidable conclusion that among the stars of nearly common motion, and thus of low velocity dispersion relative to one another, there will often form accidentally more or less permanent associations and agglomerations, ultimately leading to the formation of vast semi-permanent "local systems," as well as of real star-clusters of a more permanent kind. The stars surrounding the agglomerations, with velocities relative to these sufficiently large to prevent their absorption as members, will often suffer considerable perturbations in their orbits when passing by.

In the case of large relative velocities we may with some confidence predict the "type" of a disturbance. The disturbing cloud will then in all probability follow a circular motion in the galactic plane far closer than the disturbed particle does. We may accordingly assume the cloud to be at rest in a co-ordinate system which follows a certain circular motion of angular velocity ω_c. The Jacobi integral for the motion of the disturbed particle can now be written

$$\Pi^2 + (\Theta - \omega_c R)^2 + Z^2 - 2(V + \tfrac{1}{2}\omega_c^2 R^2) = \text{const.}, \tag{14}$$

or

$$I_1 - 2\omega_c I_2 = \text{const.}, \tag{15}$$

as we find easily by inserting the expressions for I_1 and I_2. The type of the disturbance is thus

$$\frac{\Delta I_1}{\Delta I_2} = 2\omega_c, \tag{16}$$

i.e. the displacement in the I_1, I_2 diagram proceeds parallel with the tangent between characteristic parabola and envelope. The corresponding recoil effect will tend to press the large condensations inwards into circular orbits

of smaller I_2, and thus of smaller radius R and higher angular speed ω_c than before.

The disturbances of stars of fairly high velocities relative to the circular motions should be expected to be most frequent along the inner part of the orbits ; thus near $R_{\min}$ of an orbit, where the star passes a long way through fairly dense strata which are likely to contain very massive condensations. We must then draw the conclusion that the "high velocity" stars spread from inner to outer regions of the system with, approximately, a type of disturbance $\frac{\Delta I_1}{\Delta I_2}$ equal to the value of $2\omega_c$ near $R_{\min}$ of the orbit. As to the observed "high velocity" stars in our neighbourhood, we must assume these to have emanated largely from regions situated somewhat closer to the nuclear regions than we are ourselves. The consequence would be that the "high velocity" vectors appearing at our point, which lies near the galactic plane, ought to show a strong tendency *to run parallel with the galactic plane*; in other words, that we must expect the dispersion in Z_0 to be small. We have a beautiful accordance with this conclusion in Oort's * and Raymond and Wilson's † empirical results, which show a velocity distribution of very considerable flattening towards the galactic plane, in the sense that the dispersion in Z is found much smaller than that in Π. The strong *preferential motion* of these stars along the radius vector, *i.e.* the high dispersion in Π compared with the dispersion in the Θ direction, is of course to be expected as an effect of the curvature of the envelope, *i.e.* the decrease of ω_c with increasing R, in the ordinary way, as explained above.

Passing now to smaller relative velocities, which approach to the velocities of the condensations themselves relative to the circular motions, we must conclude that the resulting disturbances are more likely to be of a random type, which no longer follows the Jacobi integral. For a certain ΔI_1 we may therefore expect a range of different values of ΔI_2. Considering the sum $\Sigma\Delta I_1$ for the net stream at a certain level of I_1, we must expect the average type $\frac{\Delta I_1}{\Delta I_2}$ to have a value considerably exceeding $2\omega_c$ at $R_{\min}$. Indicating the schematical course of average "type" in relation to "level" in the I_1, I_2 diagram, we therefore get a curved line (dotted lines in fig. 2) which in its lower part proceeds at high inclination against the envelope E, but in its upper part assumes a course corresponding to a coefficient of inclination equal to the value of $2\omega_c$ near $R_{\min}$ for the combinations of I_1 and I_2 in question. When taking all disturbed and disturbing matter together, the mean value of I_1 must be constant. We therefore conclude that, considering the net stream of particles over fixed levels in I_1, for low levels the recoil effects towards decreasing I_1 must predominate, and the net stream in question must occur in this direction. We can then understand from the run of the lower parts of the curves that the recoil effect by the outgoing net stream of particles will include a packing of matter towards the envelope,

* *Groningen Publ.*, No. 40, 1926 ; *B.A.N.*, **4**, 269, 1928.
† *A.J.*, **40**, 121, 1930.

i.e. a decreasing dispersion of the velocities relative to the circular motions for the remaining matter.

We may presume that also in the case of the smaller relative velocities the scattering effects will have a tendency to increase in intensity with decreasing R, *i.e.* with increasing density of matter, and that the scattered stars in our neighbourhood will emanate largely from somewhat deeper regions in the system. This means, however, that also in this case the velocities relative to the centroid at our point of the system will show preference for the galactic plane in the way that the dispersion in Z will be smaller than that in Π, though the effect must be expected to be less accentuated than in the case of the "high velocity" stars. We can thus account qualitatively for the *third axis* in the velocity ellipsoid at our point of the

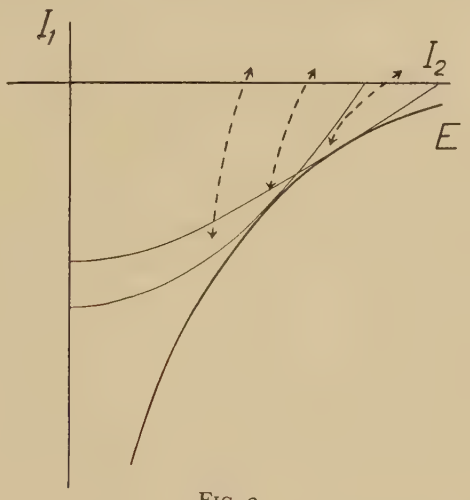


FIG. 2.

system; the ratio between the two galactic axes is given by (13). The observed deviation of the vertex of the general velocity ellipsoid (or of the line connecting the true vertices of Kapteyn's star streams) of about 15° from the radius vector is probably of an accidental nature, as is indicated by the fact that the vertex direction shifts steadily with increasing velocity dispersion towards the theoretically predicted position (*cf.* Raymond and Wilson, *loc. cit.*). For the "high velocity" stars the agreement is very good.

Any attempt to obtain a quantitative estimation of the recoil effect in the velocity dispersion must be very conjectural in the present state of our knowledge. Let us assume, however, to fix our ideas, that the condensations which are most effective in spreading the "high velocity" stars are clouds of mass 10^6 solar masses and of radius 100 parsecs, thus rather loose formations with a density of only one solar mass per four cubic parsecs. We find, however, that even such loose clouds are able to deflect relative velocities of 60 km. per sec. through several degrees. Taking the average increase of velocity necessary for ejection to be about 60 km. per sec. in the Θ direction, and assuming that a star meets a few large condensations during each revolution of the system, we may estimate that among the stars with

velocities relative to the centroid exceeding 60 km. per sec. perhaps one per cent. become ejected during one revolution of the system, *i.e.* during a time of $2 \cdot 10^8$ years. Assume now that the high velocity stars in question form about $\frac{1}{10}$ of the total mass at our neighbourhood, and that their mean velocity relative to the centroid is about 100 km. per sec. The lost "high velocity" stars are supposed to be steadily supplied from the matter of lower dispersion relative to the circular motions, in the way just explained, and the decrease of the mean square of the centroidal velocities in our neighbourhood in consequence of the ejection should be about $10^{-3}(100)^2$, or 10(km. per sec.)^2.

In this way we should expect a reduction in the general velocity dispersion by a few km. in 10^9 years. It appears from this that the effect is fairly small for the present state of our system. On the other hand, it is probable that the process of ejection has been much more violent at earlier stages of development of our system, and that the phenomenon of the Milky Way, the immense concentration of stars towards the galactic plane, is to a considerable extent the product of a process of this kind. Accepting an analogy between the motions of the stars in the stellar system and the motions of the molecules in a gas, we may say that the Milky Way is an effect of the cooling of the stellar gas by the evaporation of its swift molecules.

It is, nevertheless, not out of the question that processes of a different kind have given important contributions towards the same end. As explained in a previous paper, there are some indications that our planetary system has condensed out of a sort of "meteoric" cloud, and we may perhaps assume that in general a condensation of gaseous matter in space produces in the first instance small particles, which, however, when they appear together with the gaseous matter in real cloud formations, may combine into compact bodies of large size. We may consider here briefly a few points concerning the production of such particles. In a state of the stellar system similar to the present one a condensation of matter into meteoric particles appears to be a necessary consequence of the very low temperature of such particles, on account of the low energy density in interstellar space, in contrast to the high temperature of the surrounding interstellar gas. Meteoric particles are thus likely to grow in mass by a sublimation of matter on their surface. The energy of impact of an atom striking the surface layers of the particle is taken up by the molecules bound in these layers and is rapidly transmitted to space in the form of radiation. If we assume with Eddington * a temperature of the interstellar gas of 10,000°, the average velocity of the gas molecules for atomic weight 50 will be about 2 km. per sec. If the gas-density, according to Gerasimovič and Struve,† corresponds to 10^{-26} grm. per cm.^{3}, and if we assume a specific weight of the meteoric material equal to 5, the time necessary to form particles of the mass m grm. from an "infinitesimal" nucleus ought to be roughly

$$t = 10^{14} m^{\frac{1}{3}} \text{ years.} \qquad (17)$$

Thus particles of mass 10^{-15} grm. should have had time to form in 10^9 years.

* *The Internal Constitution of the Stars*, p. 373, 1926. † *Ap. J.*, **69**, 7, 1929.

This is about the mass of the particles in the obscuring clouds of the Milky Way according to Schalén's* results. The density of these clouds is further found to be of the same order of magnitude as the value 10^{-26} grm. per cm.^{3} assumed above. The "interstellar" particles formed in this way are likely to possess on the average small velocities relative to the circular motions and a much greater galactic concentration than the interstellar gas itself. This applies also to occasional cloud formations of gaseous and meteoric matter of higher density. In the outer regions of such formations the growth of the meteoric particles should be expected to be rapid in proportion to the density, so that for a density 10^{-22} grm. per cm.^{3} (corresponding to about one solar mass per cubic parsec or one atom per cm.^{3}) particles of 10^{-3} grm. will form in 10^9 years. This seems to be near the minimum mass of observed meteors. In much denser regions very massive particles may form, thus at density 10^{-15} bodies of mass 10^{18} grm. in 10^9 years, and these may, moreover, occasionally combine into still larger bodies. On the other hand volatilization due to violent encounters between meteors must become frequent. As temperature and density will increase under contraction, the meteoric material near the centre of the cloud may volatilize entirely, and we get *a meteoric cloud surrounding a dense gaseous nucleus* as a possible type of formation at the side of the widespread, extremely thin "obscuring clouds." Both types may be thought to develop at the present state of our system, and we may venture the hypothesis that the condensed type, with a nucleus of high temperature, really represents a star in its primordial state. The extremely strong galactic concentration of certain types of stars, O and B stars, δ-Cephei stars, etc., suggests that these objects, as well as the central obscuring stratum of small particles, have actually formed during what we may name the "*present phase*" of our system. This term may be defined by the present values of the fundamental properties of the system, such as the total angular momentum and the plane of rotation, allowing for the possible gradual changes in these qualities in consequence of the ejection of particles discussed in the preceding parts of the present paper. The minimum length of time passed in the "present phase" may probably be set down to about 10^{10} years. However, it does not seem necessary to make off-hand the same assumption concerning the stars of the system in general, and it seems quite possible that the system in its "present phase" has always been composed essentially of stars.

Concerning the original state at the beginning of the "present phase" of our stellar system we are, of course, very much in the dark. Perhaps, as suggested by the present author,† the agglomeration of two systems which after repeated "inelastic" encounters finally settle down into a single unit, may be responsible for the high angular momentum. Some interesting points bearing on the question regarding the origin of the rotation of the stellar system have recently been advanced by Strömberg.‡

* *K. Svenska Vetenskapsakademiens Handl.*, **13**, No. 2, 1934 (*Upsala Medd.*, No. 58).

† *Arkiv f. Matematik, Astronomi och Fysik*, **21**A, No. 3, 1928 (*Stockholm Medd.*, No. 1).

‡ *Mount Wilson Contr.*, No. 492 = *Ap. J.*, **79**, 460, 1934.

We proceed lastly to discuss briefly the partial instability of the system which, at least to some extent, may be a consequence of the increased flattening of the outer regions and of the proceeding general concentration of the system. If the density of matter in the disk begins to decrease rapidly at a certain R, the envelope curve in the I_1, I_2 diagram will no longer proceed regularly in its upper course. If the decrease of potential is sufficiently rapid, the value of I_2 for the circular motions will for a certain interval in R, between R_1 and R_2, say, decrease with increasing R. The envelope will

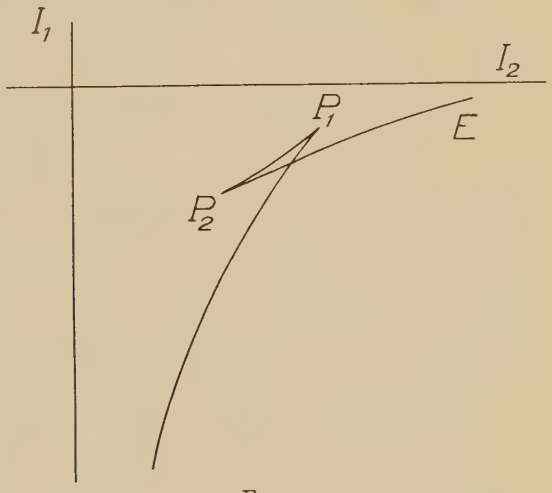


Fig. 3.

then possess two turning-points, P_1 and P_2 in fig. 3, corresponding to the limits of the interval between R_1 and R_2.

The condition for the appearance of this phenomenon is as follows. We define $u = \frac{1}{R}$ and form, according to well-known principles in the theory of central forces,

$$\left(\frac{du}{d\theta}\right)^2 = \psi(u).$$

For a point on the envelope, *i.e.* a circular motion, we have

$$\psi(u) = 0, \qquad \psi'(u) = 0.$$

The condition in question then becomes

$$\psi''(u) > 0, \tag{18}$$

or

$$\frac{\partial^2 V}{\partial R^2} + \frac{3}{R}\frac{\partial V}{\partial R} > 0, \tag{19}$$

as can be seen also by forming directly the quantity $\frac{dI_2}{dR}$. We know * that

* Cf. *K. Svenska Vetenskapsakademiens Handl.*, **4**, No. 7, 1927 (*Upsala Medd.*, No. 31).

this condition is fulfilled at the edge of a spheroidal disk of an eccentricity of the meridian section exceeding 0·834, and for somewhat higher eccentricities, if an additional central mass is assumed. It is extremely probable that the phenomenon in question (or at least a very sharp bend of the envelope, which will to some extent have the same consequences) occurs at a certain stage in the development of a very flattened system. In the case of our system it seems even possible that a discontinuity or sharp bend of the envelope may occur *between us and the centre in Sagittarius*. It is now easily seen that the matter which lies below the curve P_1P_2 in the diagram will

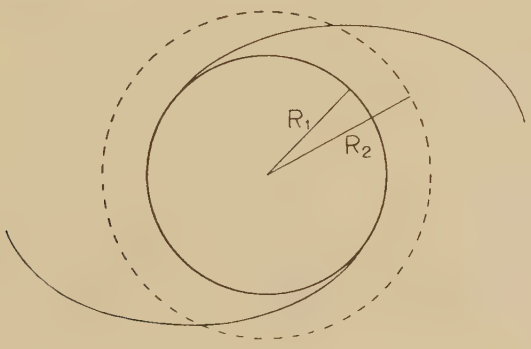


Fig. 4.

remain in a closed "inner" formation, while as soon as a particle traverses P_1P_2 it will suddenly be ejected to much larger distances, corresponding to the characteristic parabolas of the part of the envelope which proceeds from P_2 outwards. Inner disturbances leading to a deflection of particles above P_1P_2 will thus lead to the appearance of spiral orbits proceeding outwards from the radius R_1 (fig. 4).

In many cases the motion in these orbits will be quite at random, so that a point in the plane of the motion for which $R > R_1$ will be traversed in both directions, inwards as well as outwards, by matter following the orbits in question. Then there will be no visible discontinuity in shape of the disk in consequence of this motion. It seems quite possible, for instance, that the Taurus and Ursa Major streams at our point of the galactic system follow orbits of this kind, if our place in the system lies outside of R_1. Such a condition would help to explain the general flattening of the observed velocity-distribution towards the galactic plane, in agreement with the point of view suggested earlier in the present discussion.

In certain cases, however, the tidal forces of the ejected matter may be strong enough to cause a continuation of the process. In such a case these tidal forces are likely to cause the points of ejection on the circumference to recede rapidly in a direction opposite to the direction of the rotation, so that the points of ejection get more or less fixed in space. An examination of a typical spiral, *e.g.* the nebula M81, shows, however, that the ejection points have not been steadily at rest, but have moved more or less accidentally during the process.* This is entirely in agreement with our general

* *Loc. cit.*

ideas on the subject, which give room for the obviously rather capricious nature of the spiral phenomenon. To follow in detail the way in which the disturbance eventually proceeds inwards, shearing the thin strata of the disk into a more or less plain spiral structure, is a task which appears to present great difficulties and will not be attempted here.

Stockholm Observatory,
Saltsjöbaden :
1934 *October.*

THE RECENT ECLIPSE OF ζ AURIGÆ.

Arthur Beer, Ph.D.

(Communicated by the Director of the Solar Physics Observatory, Cambridge)

ζ Aurigæ is one of the most interesting star-systems. A K5 supergiant, the primary, and a B2 star, the secondary, form a spectroscopic binary of which the orbit has been determined. Subsequently to its discovery as a spectroscopic binary the system was recognized as being also an eclipsing binary. The combined spectroscopic and photometric observations were completed by observations in the atmosphere of the K star, interpreted as a rotation effect. The case is quite similar to that of Algol, and the system can be completely determined. The most interesting and important point is that such a determination of diameter and mass of a red supergiant will, when sufficiently accurate data are available, be the first we possess based on pure observation and independent of hypothetical assumptions.

The problem of ζ Aurigæ is first indicated in Miss Maury's description: "This star is one of the most beautiful examples of composite spectra found among stars not known to be double." * Eleven years later W. W. Campbell announced that the radial velocity of the star was variable. In 1924 an exhaustive study was made at the Victoria Observatory by Harper, which resulted in obtaining the orbital elements.† The majority of his spectrograms gave the appearance of a K-type spectrum with almost continuous bands superimposed. The absorption lines were diffuse and broad. But one of his plates showed sharp intense lines, very different from the others. This plate was taken when the primary was closest to the earth, and the secondary was presumably eclipsed by the primary, leaving a single K spectrum. In 1926 K. F. Bottlinger ‡ at the Berlin-Babelsberg Observatory drew special attention to this, and predicted the middle of the next eclipse for 1932 January 22. Professor P. Guthnick arranged at his Observatory, during the critical time 1931 December to 1932 March, a series of photometric and spectroscopic observations, and H. Schneller actually detected the decrease of light as predicted by Bottlinger. Guthnick's very interesting

* *Harv. Ann.*, **28**, 99, 1897.
† *Pub. D.A.O.*, **3**, 151, 1924.
‡ *A.N.*, **226**, 239, 1926.

reports * † on this eclipse led to the provisional derivation of the dimensions of the system, using as rotational effect an observed shift of the [K] line during the time immediately after the end of the eclipse. In addition, Professor J. Hopmann † ‡ in Leipzig made further photometric, and especially colorimetric, observations, and combining them with Guthnick's spectroscopic results he gave for the dimensions of the orbit, the masses, the diameters and the densities :

$$a_K = 3 \cdot 10^9 \text{ km.}, \quad a_B = 7 \cdot 10^9 \text{ km.}; \qquad M_K = 29\odot, \quad M_B = 12\odot;$$
$$R_K = 335\odot, \quad R_B = 2 \cdot 4\odot; \qquad d_K = 0 \cdot 8 \times 10^{-6}, \quad d_B = 0 \cdot 9.$$

The year 1934, with the next eclipse predicted for August–October, brought a very interesting opportunity to test the different assumptions on which all these results were based, results very much affected by incomplete observations.

In England ζ Aurigæ was included by the Solar Physics Observatory at Cambridge in its spectroscopic programme, and by the University Observatory, Cambridge, in its photometric programme. The star was observed also § at the Berlin-Babelsberg Observatory, both spectroscopically and photometrically, also at Königsberg, Potsdam, Stuttgart, Wolfersdorf, Wilno, Kiel, Victoria and Mt. Wilson.

Concerning the photometric data, it was very fortunate that at the Observatories at Cambridge and Königsberg some observations were obtained which fell just in the few hours of light-decrease on August 24. At most of the above-mentioned observatories it was cloudy. And also for the end of the eclipse Dr. K. Walter of Königsberg was successful in obtaining observations of the last part of the light-increase.

I am very much indebted to Professor Guthnick, who has shown a kindly interest by writing to me about the observations, and I am also much obliged to Dr. W. M. Smart and Mr. H. E. Green at Cambridge, and Dr. K. Walter at Königsberg, for their friendly communications of the data obtained by them in advance of publication.

We are interested only in those few photometrical data which allow us to derive with a very satisfactory accuracy some elements of this last eclipse.

Dr. Smart and Mr. Green are giving particulars of their photoelectric observations in the following paper. Dr. Walter (Nos. 3 to 9) observed the star with the 30-cm. refractor at Königsberg in normal photographic light. Each of his plates contains four intrafocal images of ζ Aurigæ, which are compared by means of an objective grating with *e* Persei and λ Aurigæ. The measurements were made with a new Zeiss photoelectric photometer, which gives for each set, according to Dr. Walter's communication, an accuracy of $0^m \cdot 01$ to $0^m \cdot 02$.

Using the observations 3 and 4 and assuming a constant rate of

* *Sitz.-Ber. d. Pr. Akad. d. Wiss., Berlin*, Phys. Math. **10**, 1932.

† *A.N.*, **245**, 9, 1932.

‡ *Ber. d. Math. Phys. Kl. d. Sächs. Akad., Leipzig*, **85**, 1933.

§ *Beob.-Zirk. der A.N.*, Nos. 26, 27, 28, 29, 32, 33 (1934); *A.N.*, **253**, 359, 1934.

TABLE I

Observations of Magnitude

No.	Observatory	J.D.	$-\Delta m$
			m
1	Cambridge	2427673·616	0·206
2	...	.....*	0·448
3	Königsberg	673·510	0·24
4	...	673·592	0·31
5	...	674·515	0·75
6	...	710·427	0·76
7	...	712·421	0·17
8	...	712·520	0·08
9	...	718·400	0·00

change of magnitude we can extrapolate for the beginning and end of the period of light decrease, which lasted $0^d{\cdot}879$ from J.D. 2427673·229 to 2427674·108. This is very happily checked by the Cambridge observation 1, which was made $0^d{\cdot}387$ after the beginning of the decrease. At this moment the amount of the decrease on the Königsberg scale would be $0^m{\cdot}75 \times 0{\cdot}387/0{\cdot}879$ or $0^m{\cdot}33$; the observed decrease at Cambridge $0^m{\cdot}206$ reduced to the Königsberg scale gives $0^m{\cdot}206 \times 0{\cdot}75/0{\cdot}448$ or $0^m{\cdot}34$. We can therefore fix the short descending branch of the light curve with confidence, deriving for the middle of the decrease J.D. 2427673·67. For the ascending branch we have observations 7, 8. A similar extrapolation shows that the increase lasted $0^d{\cdot}84$ from J.D. 2427711·77 to 2427712·61. We take as the time of disappearance and of emergence for the B star the mean of the two values, or $d = 0^d{\cdot}86$, and for the mid-points of the decrease and increase of light J.D. 2427673·67, 2427712·19 respectively, and for the mid-point of the total eclipse J.D. 2427692·93. Fig. 1 illustrates the conclusions as to the nature of the eclipse.

The result obtained above for the duration of increase and decrease respectively, with a mean of $d = 0^d{\cdot}86$, is a larger value than was deduced by Guthnick (*loc. cit.*, p. 5) from the 1932 eclipse, *i.e.* $0^d{\cdot}5$ to $0^d{\cdot}6$.

Using this new value we are enabled to make an independent estimation of the radius of the B star, assuming Hopmann's values (*loc. cit.*, p. 136), which are, of course, based on Guthnick's spectrographic results—taking the mass-ratio $1/\alpha = m_K/m_B = 2{\cdot}36$, inclination $i = 77°{\cdot}5$, and Harper's orbital element $a_K \cdot \sin i = 294{\cdot}10^6$ km. We find the approximate relation :

$$R_B = d \cdot a_K \cdot \pi \cdot (1 + \alpha)/\alpha P$$

or

$$R_B = 2{\cdot}9 \times 10^6 \text{ km.}$$

Ignoring uncertainty arising from the curvature of the orbit over a longer interval and from the unknown position of the projection of the orbit of the B star on the disc of the K star, we can similarly calculate from the length of

* Mean of observations during totality.

totality of the eclipse the approximate diameter of the K star. This comes out at $R_K = 130 \times 10^6$ km. It must be remembered that both these diameters depend upon Guthnick's value for α.

For the spectrographic observations at the Solar Physics Observatory at Cambridge the 2-prism spectrograph attached to the Newall telescope was

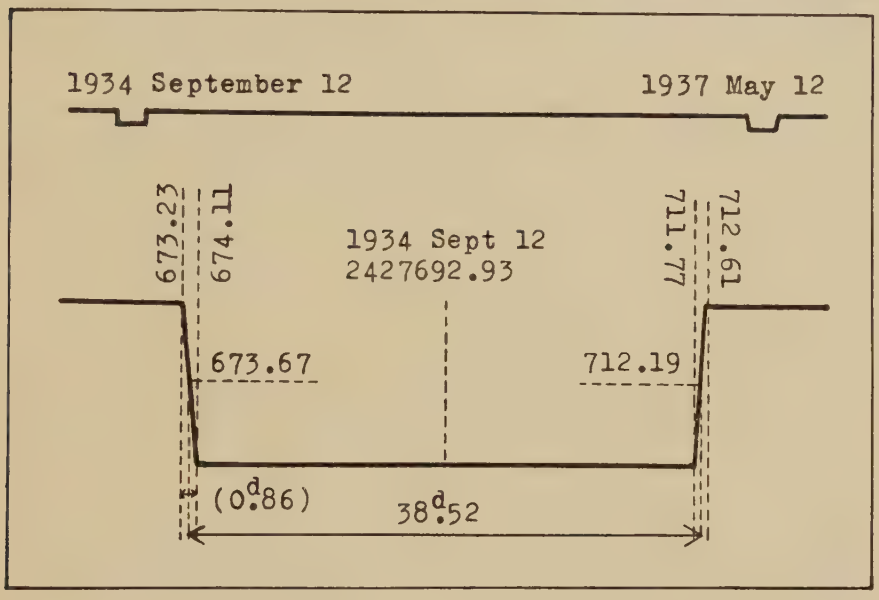


FIG. 1.—*Details of the Eclipse of ζ Aurigæ.*

adjusted for the [H] and [K] region.* Professor Stratton and his colleagues obtained spectrograms with exposures ranging from 3 to 5 hours on the following nights.

TABLE II

Dates of Spectrograms

Plate No.	Time of Mid-exposure	J.D.
	h m	
355	Sept. 26 0 42	2427705·530
356	27 1 25	706·559
358	29 1 45	708·573
359	30 2 00	709·583
360	Oct. 5 1 00	715·542
361	6 1 27	716·560
362	9 1 52	719·578
363	10 0 57	720·540
364	12 1 27	722·560
367	24 0 38	734·526

* To avoid confusion between the [K] line and the K star this notation recommended by the I.A.U. is here adopted.

Unfortunately the emergence of the B star from eclipse took place on J.D. 2427712, in the middle of a spell of cloudy weather.

As comparison spectrum an iron spark was used. For the spectrophotometric investigations standard comparison plates were secured as usual: the plates were cut in half, one half being used for the star and the other half for a standardised wedge spectrum; the two halves were developed together.

The plates 355 to 359, which were taken during totality, show no measurable details in the region [H] to [K], even with an exposure of 5 hours on September 29 and of 4 hours on September 30. On plate 360, 3 days after the end of the eclipse, exposed for only 3 hours, the continuous spectrum due to the B star is strong over the range [H] to [K], and the most striking feature is a very intense narrow absorption line at [K]. On plate 361 on the following day this [K] line is much fainter, and it decreased into insignificance on plates 362, 363, 364. On plate 367, 22 days after the end of the eclipse, the [K] line cannot be traced (see fig. 2, Plate I).

The strong narrow absorption [K] line is caused by the light of the B star going through the atmosphere of the K star. In order to obtain the displacement of the [K] line from the position that it would occupy in the reversing layer of the K star and to deduce the velocity of rotation of the chromosphere of the K star, we have first to determine the radial velocity of the K star.

In Harper's elements of the spectroscopic orbit we have to correct the value of T, taking into account the recent photometric observations. His value of $T = 2415122$ gives the middle of the eclipse at 2427702 instead of the observed value 2427693. Correcting T by -9^d we get $T = 2415113$. With this value of T and Harper's other elements, $P = 973^d$, $\epsilon = 0{\cdot}41$, $\omega = 330°$, $K = 23{\cdot}8$ km./sec. and $\gamma = +10{\cdot}7$ km./sec., we obtain for the radial velocity of the K star on the days of the various plates the following figures :—

TABLE III

Radial Velocity of the K Star

Plate No.	V_K in km./sec.
360	+26·3
361	+26·6
362	+27·6
363	+27·8
364	+28·5

Table IV below gives the observed radial velocity given by the [K] line, $V_{[K]}$, reduced to the Sun, and the difference between this velocity and that given by the K star.

The third column of figures in Table IV confirms the presence of the rotational effect examined by Guthnick in the 1932 eclipse. Negative values are just what is to be expected if the direction of rotation of the K star is in the same sense as the orbital motion.

A future discussion of all the available material of this last eclipse should help to decide whether any extrapolation can be made from the observed

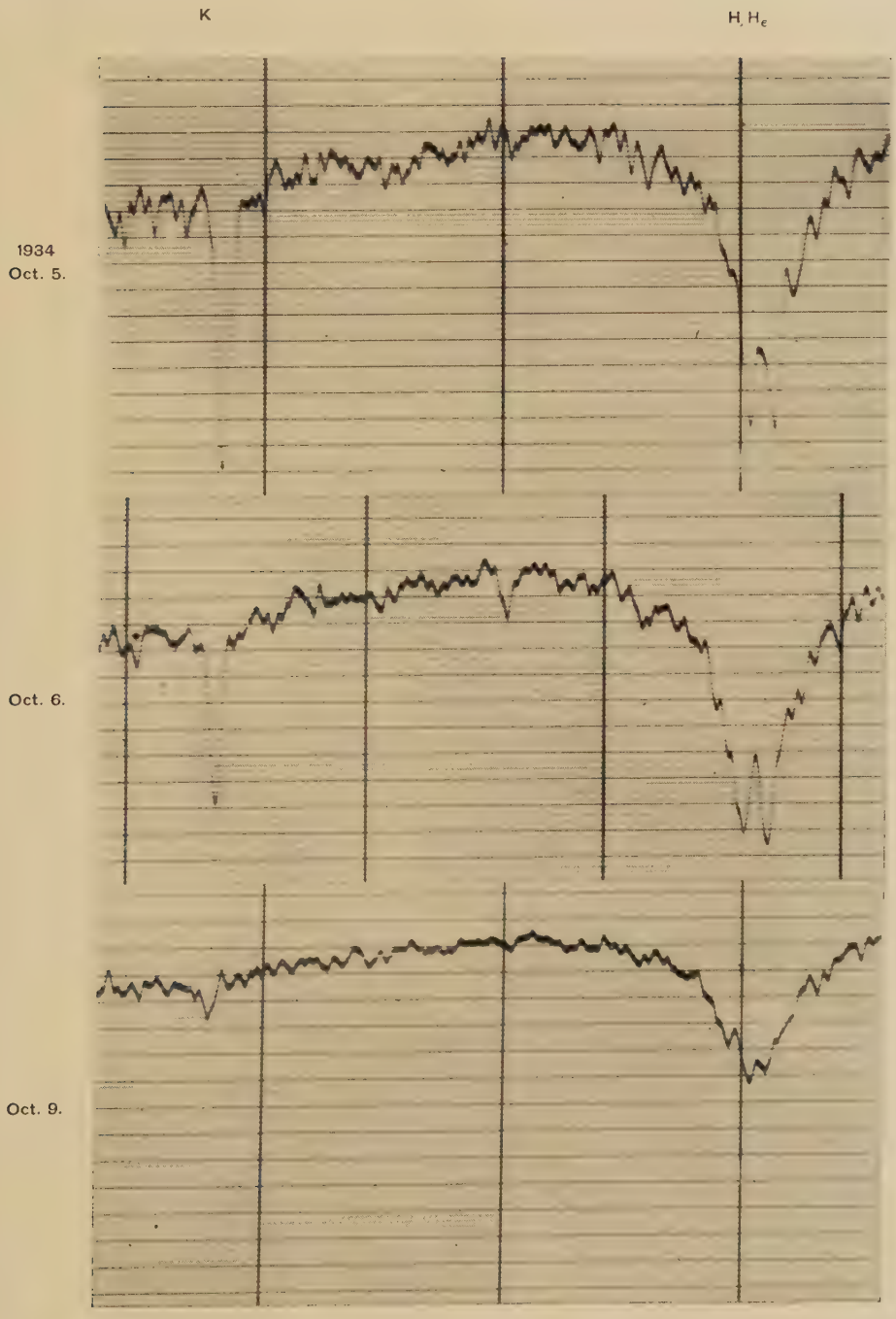


Fig. 2.—Spectrophotometer Tracings of the Region K to H_ϵ of ζ Aurigæ.

TABLE IV

Velocity given by the [K] Line

Plate No.	$V_{[K]}$	$V_{[K]} - V_K$ in km./sec.
360	+ 5·3	−21·0
361	+ 9·4	−17·2
362	+ 1·5	−26·1
363	+ 0·7	−27·1
364	+13·7	−14·8

values of $(V_{[K]} - V_K)$ to values at the end of the eclipse, such as Guthnick made for the 1932 eclipse. It cannot be done from the present data. It would seem that the spectral conditions in the chromosphere of ζ Aurigæ at the recent eclipse differ from those holding at the 1932 eclipse.

This appears also in the changes of width and intensity of the [K] line. On plates 360 and 361 of the Cambridge spectrograms, 3 and 4 days respectively after the end of the eclipse, the width of [K] is 1·36 A. and 0·92 A. On later plates the line is too weak for accurate measurement of its width. From the eclipse data in 1934 we can fix the date of emergence of the B star from eclipse in 1932 as February 1. On February 5 Guthnick gave the width of [K] as 1·1 A., and on February 15 as 0·7 A.

The date of complete disappearance of the [K] line was uncertain over a range of 12 days in 1934 and of 10 days in 1932 owing to spells of bad weather. But we may notice that [K] was visible for at least 34 days after eclipse in 1932 and for not more than 22 days in 1934. That the increase in intensity took place in 1934 much faster than in 1932 strongly suggests changed conditions in the chromosphere. The height of the Ca^+ chromosphere, of sufficient density to show the [K] absorption line, lies (with the same uncertain factors as in the calculation of R_K on p. 27) between 70×10^6 km., and 90×10^6 km.

The [K] line has also been examined spectrophotometrically. I am much indebted to Mr. E. G. Williams for the preparation and reduction of the records obtained by the recording microphotometer of the observatory (described by J. A. Carroll and E. B. Moss in *M.N.*, **91**, 191, 1930). Table V gives the total absorption, T, and the depth, D, of the [K] line for the plates 360 to 364. The total absorption has been measured allowing for the [K] line in the reversing layer of the K star, which causes a shallow depression on the tracing. The total absorption of the [K] line in the chromosphere of the K star is equivalent to the complete absorption of T angstrom units of the continuous spectrum, which is, of course, almost entirely due to the B star. The value of D, which is given as a percentage of total absorption (= 100), is mainly decided by the various optical and photographic factors involved and has no direct physical significance.

The last column of Table V gives the interval in days from the end of the eclipse to the time of the plate. Figs. 3 and 4 give, in graphical form, the values of Table V.

One is tempted to attempt to determine the law of density of the Ca^+

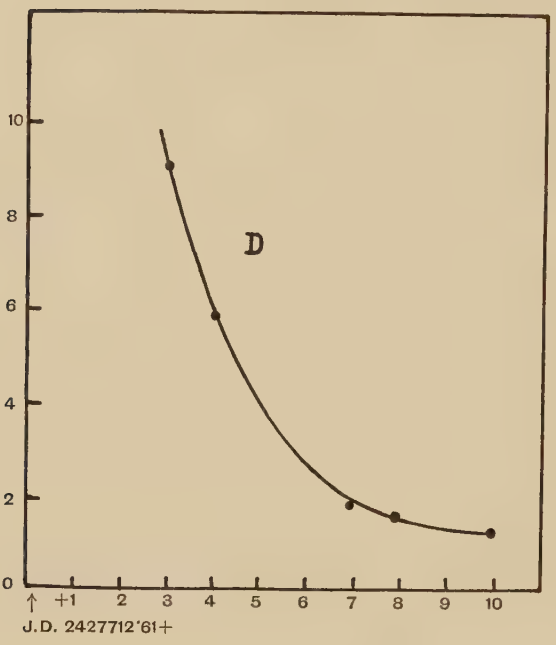


FIG. 3.—*Depth of the [K] line of ζ Aurigæ.*

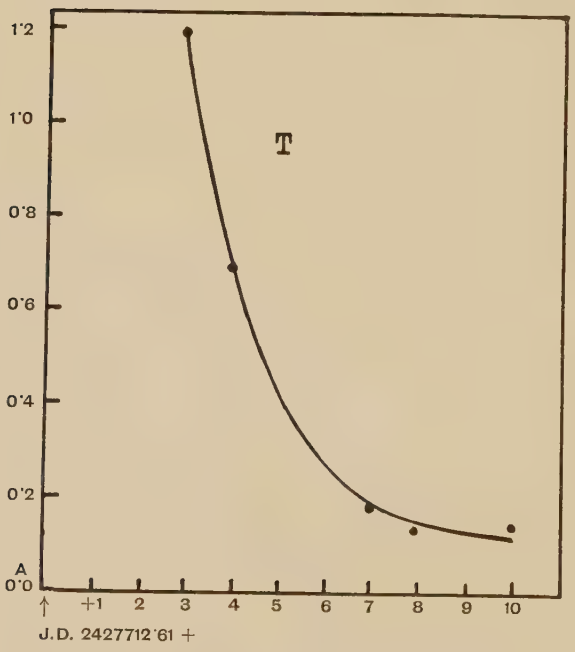


FIG. 4.—*Total Absorption of the [K] line of ζ Aurigæ during the first ten days after the end of the eclipse.*

TABLE V

Absorption of [K] Line

Plate No.	T	D	t d
360	1·19	90	2·9
361	0·68	58	4·0
362	0·18	18	7·0
363	0·14	16	7·9
364	0·15	12	10·0

chromosphere from the rate of fall of the absorption. But the data available are too scanty and too many unknown factors are still involved. It seems clear that no uniform distribution of density will account for the observed results and still less can a thin shell do so. The law of decrease of density with height may be calculable when more extensive data become available.

It is my pleasant duty to express my sincere thanks to Professor F. J. M. Stratton for his very kind permission to use the material of the Solar Physics Observatory, and also for his helpful suggestions.

Summary.

(1) From photometric observations at Cambridge and Königsberg the details of the 1934 eclipse of the B component of ζ Aurigæ have been obtained.

(2) Spectrographic observations show that for at least 10 days after emergence from behind the K component the light from the B star passes through an extensive chromosphere of Ca^+ belonging to the K star.

(3) The displacement of the sharp [H] and [K] lines from the positions appropriate to the K star give a rotational value for the chromosphere of about 19 km./sec. This depends upon Harper's orbit.

(4) The rapid fall of intensity of the [K] line shows that the conditions of the chromosphere of the K star are not the same as at the eclipse of 1932.

PHOTOMETRIC OBSERVATIONS OF ζ AURIGÆ.

W. M. Smart and H. E. Green.

(Communicated by the Director, Cambridge Observatory)

The components of this remarkable eclipsing binary are a supergiant K5 star and a B2 star, their radii,* in terms of the solar radius as unit, being about 330 and 2·4 respectively. The period is 973 days. Further details of the system will be found in the preceding paper † by Dr. Beer. An eclipse of the B2 star occurred in 1934 August and, as the next observable eclipse will not take place till the end of 1939, the following observations

* J. Hopmann, *Abh. d. Math. Phys. Kl. d. Sächs Akad., Leipzig*, 85, 136, 1933.

† *M.N.*, 95, 24, 1934.

are now published in the hope that they will be of service to investigators of this interesting system.

The photo-electric cell in use is a potassium cell filled with argon. η Aurigæ, spectral type B3 and visual magnitude 3·28 (Harvard scale), was the comparison star. A set of two observations, for example, was made in the following order: η, ζ, η, ζ, η, and it is the means of the magnitude differences for the several observations of a set, in the sense

$$\Delta M = m_\zeta - m_\eta,$$

that are given in the following table, the corrections for atmospherical extinction having been previously applied. Owing to the close proximity of η and ζ in the sky, these extinction corrections are extremely small; the largest is only $0^m{\cdot}007$.

Values of ΔM

Observation Number	Date of Observation	Julian Date (heliocentric)	No. of Observations	ΔM	Weight	σ (unit $0^m{\cdot}001$)
	1933	2427000 +		m		
1	Dec. 3–4	410·524	3	1·833	3	4
2	9–10	416·492	3	1·826	3	15
3	10–11	417·451	3	1·875	3	10
4	13–14	420·473	2	1·839	2	10
	1934					
5	Jan. 5–6	443·438	3	1·833	2	21
6	Feb. 2–3	471·430	1	1·820	1	...
7	8–9	477·384	4	1·826	2	9
8	11–12	480·341	2	1·833	2	10
9	Aug. 23–24	673·616	2	2·039	2	2
10	Aug. 29–30	679·612	2	2·290	2	1
11	Sept. 9–10	690·580	2	2·281	2	2
12	28–29	709·505	2	2·277	2	6
13	,,	709·555	1	2·273	1	...
14	Oct. 5–6	716·552	2	1·847	2	5

The weights in column 5 were assessed according to observing conditions. In the last column σ is the average deviation of the individual observations of a set from the corresponding value of ΔM in column 4; the values of σ give some indication of the relative consistency of the observations forming the various sets.

Observations Nos. 1–8 and 14 are concerned with the total light of the system. Omitting No. 3, which appears to be discordant,* these observations give

* Hopmann (*loc. cit.*) has found small periodic fluctuations in the total light of the system.

$$\Delta M = 1{\cdot}833 \pm 0{\cdot}004.$$

Observation No. 9 has been made when the B2 star is about half eclipsed.

Observations Nos. 10–13 refer to the constant phase at principal minimum and they give as mean

$$\Delta M = 2{\cdot}281 \pm 0{\cdot}004.$$

Observatory, Cambridge :
1934 October 24.

NOTE ON THE OBSERVATION OF LATITUDE AT GREENWICH.

S. Kawasaki.

1. In the summer of the year 1932 the writer was allowed to stay at the Royal Observatory, Greenwich, to make some observations on latitude variation. At that time, by the courtesy of the Astronomer Royal, he was also given an opportunity to make a study of the effect of the wind on the latitude observations. As to the effect of the wind on the latitude observed at the International Latitude Observatory at Mizusawa, Japan, he had already made a full discussion * and found that a systematic term is probably produced by the annual variation of the elements of the wind. But he was not certain that the so-called Kimura term in the latitude observation at Greenwich is attributable to this effect, because, unlike that at the International Latitude Observatories, the Kimura term at Greenwich shows two annual maxima. However, it is now evident through the present investigation that : (*a*) there is a clear correlation between the direction of the wind and the latitude observations made at Greenwich ; (*b*) it is also possible to explain the cause of the Kimura term at Greenwich by the effect of the wind.

2. At Greenwich the observation of latitude variation was commenced in the year 1911 and carried out continuously since that time. The Cookson floating zenith telescope was used for this purpose and, contrary to the practice of the International Latitude Observatories, observations were always made photographically. The results obtained there on the variation of latitude, together with those of the constant of aberration, have already appeared in two small volumes, and the third is expected to be published soon.

As to the first problem, (*a*) of the above two, the Greenwich astronomers have already pointed out the existence of this effect. In the report † of the latitude observations made during the first seven years, they made a discussion and concluded that the correlation between the magnitudes of the plate residuals and the directions of the wind could be represented by the simple harmonic

$$+0''{\cdot}05 \cos(\theta - 45^\circ), \qquad (1)$$

* *Proc. Imp. Acad.*, 4, 191, 1928. Further results will be published in the near future.

† *Observations made with the Cookson Floating Zenith Telescope in the Years* 1911–1918. London, 1921.

where θ is the direction of the wind measured from N. through E., S. and W.

To make the results of the meridian observations as accurate as possible, researches of this kind must be not less thorough than the improvement of the observing method and the skilfulness of the observer. So the present writer attempted to reinvestigate the same problem.

The above-mentioned correlation (1) was derived from the observations made during the years from 1911 to 1918, by simply plotting the residuals of each plate on squared paper against the direction of the wind. Here the "residual" means the mean value of the mean zenith distances north of the individual star-pairs of any group, corrected for precession, nutation, aberration and parallax. Corrections for the proper motions and temperature are also applied as well as that to the mean of the group. Accordingly, if the magnitude of the observed latitude should be considered, the sign of expression (1) is to be altered. In others words, (1) means that the observed latitude assumes the smallest value when the wind is blowing from N.E., or $\theta = 45°$.

Moreover, in these residuals, there are naturally contained the group correction, latitude variation and the mean factor due to the correction to the assumed aberration constant. So, in making a new investigation on the observations made during the years from 1919 to 1927, the present writer aimed at making the results as accurate as possible, and first of all he derived the values of Δ_{pq}, whose meaning is as follows :—

When the star-group p was observed at the date q, then from the mean residual corresponding to that group and date, Δ_{pq} is obtained by putting

$$\Delta_{pq} = (\text{mean residual})_{pq} - (z_p + t_q + \alpha_{pq}x), \qquad (2)$$

where z_p is the mean group correction to the assumed star-places for that group, t_q is the value of the latitude variation corresponding to that date, α_{pq} is the mean aberration factor for this group and date and x the correction to the assumed aberration constant. Even though the accurate values of z_p, t_q and x are not known, for the present purpose those which are given in the Greenwich report * can be adopted as the most probable values. The writer, therefore, in deriving the numerical values of Δ_{pq}, used those of z_p and x without any alteration. t_q was obtained graphically by drawing a curve which represents the observed results of latitude variation, the data for drawing this having been taken from the same source.

At Greenwich the wind is observed by means of a self-registering apparatus, which is placed at the top of a tower whose height is about 40 feet, located west of the zenith telescope room at a distance of about 60 feet. By reading the records anew, the writer found the direction of the wind during the interval when the observations of latitude were actually carried out. The mean velocity of the wind at that interval was calculated as well. Then the Δ_{pq}'s were classified in eight groups according to the direction of the wind and the mean values for each group were derived. These results are given in Table I, together with their probable errors, which

* *Observations made with the Cookson Floating Zenith Telescope in the Years* 1919–1927. London, 1928.

TABLE I

Effect of Wind at Greenwich

(unit 0″·001)

Direction of Wind .	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.
Δ_{mean} . . .	+83	+47	+36	+16	−23	−40	−13	+27
Probable Error .	± 9	±11	± 5	± 6	± 7	± 3	± 3	±10

were found by calculating the deviation of each Δ_{pq} from their mean value. If the mean values should be represented by an expression of the form

$$A + B \cos (\theta - C),$$

where θ has the same meaning as that of (1), then the numerical values of

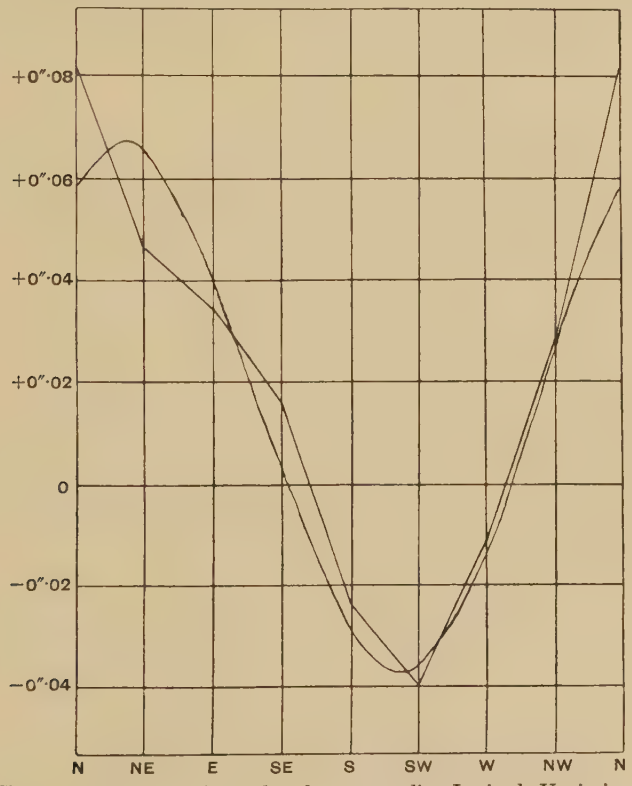


FIG. 1.—*Direction of Wind and corresponding Latitude Variation.*

these three constants can be found by giving suitable weights and forming normal equations. The solution is

$$\begin{aligned} A &= +0''{\cdot}015 \pm 0''{\cdot}003, \\ B &= +0''{\cdot}052 \pm 0''{\cdot}004, \\ C &= + \quad 34^\circ \pm 4^\circ. \end{aligned} \qquad (3)$$

In fig. 1 a curve is drawn employing these values and the broken line represents the results of Table I.

The positive value of A means that at Greenwich S., S.W. and W. winds predominate throughout the whole year. Comparing B and C with those of the expression (1), we see that they are in good agreement respectively, and these statistical investigations are enough to show beyond doubt that the latitude observations at Greenwich have a close connection with the direction of the wind.

3. Generally the mean direction of the wind makes a systematic change during the course of a year. Therefore, if the observed latitude is affected by the change of the direction of the wind, it must naturally follow that a systematic term, which is independent of the real variation of the pole of the earth, is contained in the observed variation of latitude. In the following, the Kimura term at Greenwich is considered from this point of view.

It is easily seen that the correlation between the direction of the wind and the corresponding wind factor in the observed latitude is represented by an expression of the form

$$U_i = G \cos \theta_i + H \sin \theta_i, \tag{4}$$

where θ_i represents as before the direction of the wind measured from north to eastward, by changing i from 0 to 7, so that in the case of the N.E. wind $i=1$, accordingly $\theta_1 = 45°$, and so on. Then the variation of latitude due to the variable directions of the wind during any tenth of a year is obtained by the formula

$$\mathbf{U}_j = \frac{\sum_{i=0}^{7} U_i N_{i,j}}{\sum_{i=0}^{7} N_{i,j}}, \qquad j = 0, 1, \ldots, 9, \tag{5}$$

where $N_{i,j}$ is the number of latitude observations made in the case when the wind was blowing from the direction corresponding to i during the interval of the jth tenth of a year.

From (4) and (5) we get

$$\mathbf{U}_j = GQ_j + HR_j, \qquad j = 0, 1, \ldots, 9, \tag{5'}$$

in which

$$Q_j = \frac{\sum_{i=0}^{7} N_{i,j} \cos \theta_i}{\sum_{i=0}^{7} N_{i,j}},$$

$$R_j = \frac{\sum_{i=0}^{7} N_{i,j} \sin \theta_i}{\sum_{i=0}^{7} N_{i,j}}.$$

Since G and H are both constant, (5′) means that **U** varies according to the variations of Q_j and R_j. Moreover, Q_j and R_j are the functions of $N_{i,j}$, or of frequencies of each direction of the wind. Therefore it follows that, so long as the expression (4) exists, the part of the variation of latitude, due to the variation of the direction of the wind, is simply derived by counting the frequencies of the wind for each direction. Practically, to find the accurate values of G and H is rather difficult, and accordingly the determination of the amplitude of the variation of **U** is also difficult. But as we can find the numerical values of the Q_j's and R_j's fairly accurately, the period and the phase of **U** are comparatively accurately determined, even when absolute values of G and H are lacking and only relative values are known.

Returning to the observations at Greenwich, the numerical values of $N_{i,j}$ during the years from 1919·6 to 1927·1 are shown in Table II, where the figures in the first line represent the direction of the wind and those in the first column the tenth of a year. From this table the numerical values of Q_j and R_j are derived. Their results are tabulated in Table III. Glancing at these values, it is easily recognized that both Q_j and R_j vary, exhibiting two

Table II

Numerical Value of N

	0	1	2	3	4	5	6	7
0	7	1	3	8	18	48	37	15½
1	6	6	6	12	12	58	16½	4
2	14½	11	27½	10½	15	44	10½	9
3	12½	17	16½	5½	5	27	37	3½
4	11	29½	21½	16	6½	54	24	8½
5	17	13½	27	19	11½	34	38	9
6	5	5	3	4	10	53	45	13½
7	9	7½	4	12	30½	71	33	9
8	18½	12	27½	24½	19½	44½	8	9
9	13½	13	13½	12½	15½	36½	14	6

Table III

Numerical Values of Q_j and R_j

j	0	1	2	3	4	5	6	7	8	9
Q	−0·283	−0·402	−0·175	−0·008	−0·106	−0·095	−0·233	−0·389	−0·214	−0·186
R	−0·528	−0·345	−0·037	−0·211	−0·085	−0·109	−0·597	−0·408	+0·046	−0·101

annual maxima and minima. Therefore, even if the Kimura term at Greenwich were not yet discovered, it could be anticipated that the previously mentioned systematic effect in the latitude observations made at Greenwich must introduce a variation of semi-annual type. But the Kimura term at that observatory has in fact already been pointed out, and in addition it has features similar to those anticipated. To the General Conference of the International Astronomical Union held at Leiden, Professor Kimura presented a report in which he had made a calculation of the Kimura term at

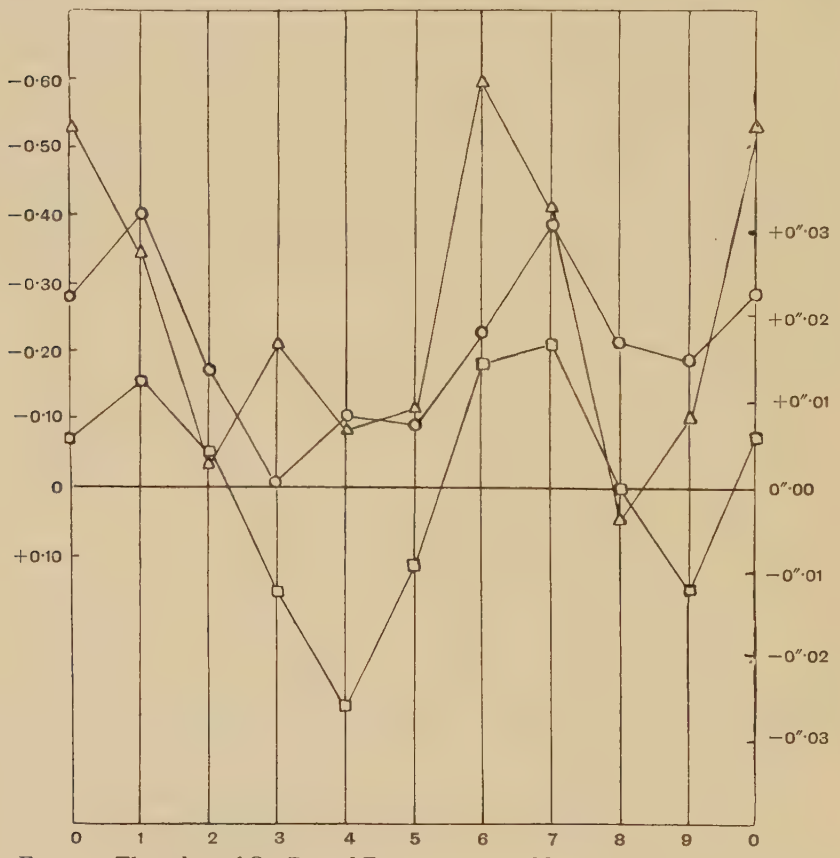


FIG. 2.—*The values of* Q_J, R_J *and* Z_G *are represented by* ○, △ *and* □ *respectively.*

TABLE IV

Kimura Term at Greenwich

(unit 0″·001)

Fraction of Year .	0·0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
Z_G . . .	+6	+13	+4	−12	−26	−9	+15	+17	0	−12

Greenwich by using the data observed during the years from 1919·0 to 1927·0. The numerical values are represented in Table IV. As the two sets of numbers given in Tables III and IV are derived from data extending over the same interval, it is interesting to make a comparison of them. Fig. 2 is drawn for this purpose. The scale for Q_j and R_j is marked at the left-hand side of the figure, and the Kimura term is measured by that on the right-hand side of it. We can see that all of these three quantities vary similarly ; the period and the phase being common to all, and the maximum values occurring near 0·0 or 0·1 and 0·6 or 0·7 of a year. Thus it is concluded that at least some part of the Kimura term, though probably not the whole of it, is due to the seasonal variation of the mean direction of the wind.

It is also reasonable to anticipate that not only in the case of the latitude, but in any other meridian observation made at Greenwich, a relation between the direction of the wind and the astronomical data exists. If so, probably this relation will be represented by a simple harmonic resembling (4). Then we can again make a supposition that in that meridian observation there must exist a term which has the same nature as the Kimura term, though the amplitude of it may be very small.

4. Let us proceed to some consideration of the numerical values of G and H. There will be no great mistake even if we derive these values directly from B and C, whose values are given in (3). And yet there remains a question about the nature of B and C. The greatest trouble is this. Both B and C were obtained by analysing Δ_{pq}, which is defined by (2), and since t_q, which we have subtracted from each of the mean residuals of the plates, is the observed variation of latitude, it must naturally contain the systematic variation of latitude, which is due to the variation of the direction of the wind or the variation of $\mathbf{U}$. In other words, from the data, from which the effect of the wind on the latitude observations is to be derived, some part of that effect is already rejected. To answer this question, the following process was adopted :—

Let $\Delta_{p,q}$ be the corrected residual corresponding to the epoch j and the wind-direction i, then we can put

$$\Delta_{p,q} \equiv \Delta_{i,j}$$

and

$$\Delta_{i,j} = \Delta + U_i - \mathbf{U}_j,$$

where Δ is a certain constant common to all $\Delta_{i,j}$. Accordingly Δ_{mean} of Table I is written in the form

$$\frac{\sum_{j=0}^{9} \Delta_{i,j} N_{i,j}}{\sum_{j=0}^{9} N_{i,j}}, \qquad i = 0, 1, \ldots 7,$$

which becomes

$$\Delta + \beta_i G + \gamma_i H,$$

where

$$\beta_i = +\frac{\sum_{j=0}^{9}(\cos\theta_i - Q_j)N_{i,j}}{\sum_{j=0}^{9}N_{i,j}},$$

$$\gamma_i = +\frac{\sum_{j=0}^{9}(\sin\theta_i - R_j)N_{i,j}}{\sum_{j=0}^{9}N_{i,j}}.$$

The numerical values of Q_j, R_j and $N_{i,j}$ being known, those of β_i and γ_i are also known, and thus eight equations of condition are obtained for eight values of Δ_{mean} in Table I. By giving suitable weights to them we get the following normal equations :—

$$\begin{aligned} &+25{\cdot}00\Delta - 1{\cdot}43G - 0{\cdot}70H = -0{\cdot}104,\\ &-\ 1{\cdot}43\Delta + 6{\cdot}80G - 0{\cdot}13H = +0{\cdot}306,\\ &-\ 0{\cdot}70\Delta - 0{\cdot}13G + 12{\cdot}01H = +0{\cdot}384. \end{aligned}$$

The solution of these gives

$$\begin{aligned} \Delta &= -0''{\cdot}0006 \pm 0''{\cdot}0024,\\ G &= +0{\cdot}0455 \pm 0{\cdot}0046,\\ H &= +0{\cdot}0324 \pm 0{\cdot}0034. \end{aligned}$$

Δ is negligibly small as expected, and from these values of G and H we can get the expression

$$U = \underset{(\pm\,0''{\cdot}004)}{+0''{\cdot}056}\cos(\theta - \underset{(\pm\,4^\circ)}{36^\circ}).$$

Though the differences are not great, these values will be more reasonable than those of B and C.

As mentioned before, if the magnitude of the observed latitude at Greenwich is to be considered, the sign of G and H should be altered, because G and H were reduced from the "residuals" of the plates. By so doing, we get the numerical values of **U** at each tenth of a year. These values are given in Table V and their probable errors are always 0″·001. Thus it is proved that more than half of the Kimura term is attributable to the effect of the wind.

TABLE V

Numerical Value of $\mathbf{U}_j$

(unit 0″·001)

j	0	1	2	3	4	5	6	7	8	9
$\mathbf{U}_j$	+30	+30	+9	+7	+8	+8	+30	+31	+8	+12

5. The present data are not sufficient to give us any definite idea on the correlation between the strength of the wind and the latitude observed at

Greenwich. Yet Table VI suggests to us that the stronger wind probably gives the greater effect on the observed latitude. In computing the results given in this table, all observations corresponding to the same direction of the wind were separated into two groups according to the strength of the wind. Different criteria were chosen for different directions of the wind, so that both of the two groups in each direction may contain nearly the same number of observations. These criteria are given in the second line of the

TABLE VI

Wind Force and Observed Latitude

(unit 0″·001)

Direction of Wind .		N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.
Criterion. . .		11	10	7	7	9	10	12	12
Δ_{mean}	Strong Wind	+110	+62	+44	+37	−16	−33	−21	+25
	Weak Wind.	+ 62	+36	+31	− 1	−29	−47	− 7	+28
Probable Error .		± 13	± 14	± 7	± 8	± 10	± 4	± 4	± 14

table, and, for example, in the case of N. wind the latitude observation corresponding to the wind force of 11 miles per hour or greater is put into the group "strong wind." Δ_{mean}'s of this table were computed in the same way as those given in Table I. Probable errors were obtained by simply multiplying those given in Table I by $\sqrt{2}$, because the weight of the results in this table is always nearly half of that of Δ_{mean}'s given in Table I.

6. The main points of the above discussion are summarized as follows :—

(i) The effect of the wind on the observed latitude at Greenwich is derived numerically from the observations made during the years from 1919·6 to 1927·1.

(ii) It is ascertained that more than half of the Kimura term at Greenwich is attributable to the effect of the wind.

(iii) Not only in the case of latitude, but in any other meridian observation made at Greenwich, we can expect a systematic term of semi-annual type to exist.

(iv) A suggestion is made that stronger wind gives the greater effect on the observed latitude.

In conclusion, the writer wishes to express his sincere thanks to Sir Frank Dyson, the then Astronomer Royal, Dr. J. Jackson, Mr. W. M. H. Greaves and others on the staff of the Royal Observatory. Through their generosity he was able to use the valuable observed results and have access to other necessary data. Particular acknowledgment should be made of the kindness of Mr. H. H. Furner and Mr. H. W. Acton, who were always helpful to him and afforded him every convenience in the course of the present investigation.

THE STELLAR CASE OF THE PROBLEM OF THREE BODIES, WITH APPLICATION TO THE ALGOL SYSTEM.

R. A. Lyttleton, B.A.

1. *Introduction.*—This paper consists of an investigation of the changes of period of a certain class of eclipsing variable stars as determined from their light curves. The particular class under consideration is that in which a close binary star system is disturbed by a third distant star. Algol is a system of this character, and the variability of the light-curve periods is well known.

The work was undertaken at the suggestion of Dr. W. M. Smart, and the writer has been fortunate to have had his guidance in carrying out the present investigation.

The variability being assumed to be due to the perturbations by the third distant star, we proceed to discover whether this factor alone would account for the change in the period actually observed. The period as understood here is obtained by finding the mean of a number of observations extending over some interval, say two or three months, and it is compared with the period obtained in a similar way at some later date. To begin with, the theoretical solution of the problem, to the necessary order of accuracy, is required, and this work is shown in the first half of the paper. The results obtained are then applied to the Algol system, whose dimensions and elements are fairly well determined.* Algol consists of a close binary star system of period roughly 2·867 days disturbed by a third star of period 1·885 years (McLaughlin gives 1·873 years). Hellerich † gives an empirical formula, based on a large number of observations, for the times of successive minima in the form :

$$\begin{aligned}&\text{J.D. } 2378497{\cdot}7588 + 2^{d}{\cdot}86731077\, n\\ &+ 0^{d}{\cdot}1266 \sin [0^{\circ}{\cdot}01786(n - 176)]\\ &+ 0^{d}{\cdot}0119 \sin [0^{\circ}{\cdot}08762(n - 188)],\end{aligned}$$

n being the number of minima after the epoch.

The problem is attacked in the same way as the lunar problem is solved in de Pontécoulant's method. The limitations imposed are almost the same as in the lunar theory, the close pair being considered as the earth-moon system and the third body is considered to be moving in an elliptic orbit about the centre of mass of the binary system. M_1 and M_2 denote the components of the binary star and M denotes the third distant star. In this case all three masses are of the same order of magnitude. The ratio of the mean motions is found from observation to be of the order of 1/100, and accordingly the ratio of the mean distances is not so small as in the

* See Eddington, *Internal Constitution of the Stars*, pp. 209, 210 (1930); and McLaughlin, *Publ. of the Observatory of the University of Michigan*, vol. vi, No. 2 (1934).

† *Astronomische Nachrichten*, 209, 227 (1919).

lunar theory. The eccentricities and inclination are considered as of order not less than 1/10. The units of mass, length and time are so chosen that the constant of gravitation is unity and the sum of the masses of the close pair equal to the unit of mass.

2. *The Force-function.*—Write $M_1M_2=r$, $M_1M=\Delta_1$ and $M_2M=\Delta_2$ for the distances between the stars and $GM=R$ where G is the centre of mass of M_1 and M_2.

Let x, y, z be the co-ordinates of M_1 with respect to rectangular axes through M_2, and X, Y, Z the co-ordinates of M with respect to parallel axes through G; then the force-function for the motion of M_1 relative to M_2 is

$$F=\frac{M_1+M_2}{r}+\frac{M(M_1+M_2)}{M_1M_2}\left(\frac{M_1}{\Delta_1}+\frac{M_2}{\Delta_2}\right).$$

3. *The Equations of Motion.*—Take the xy plane parallel to the plane of the (elliptic) motion of M relative to G. Let the projection of r on this plane be r_1, v the longitude of r_1 measured from the x-axis and s the tangent of the latitude of r above the plane xy. Then the equations of motion are

$$\ddot{r}_1-r_1\dot{v}^2=-(M_1+M_2)\frac{r_1}{r^3}+\frac{\partial R}{\partial r_1}, \tag{1}$$

$$\frac{d}{dt}(r_1^2\dot{v})=\frac{\partial R}{\partial v}, \tag{2}$$

$$\ddot{z}=-(M_1+M_2)\frac{z}{r^3}+\frac{\partial R}{\partial z}, \tag{3}$$

wherein $R=M(M_1+M_2)(M_1/\Delta_1+M_2/\Delta_2)/M_1M_2$, and is expressed in terms of r_1, v and z.

Multiply these equations by $\dot{r}_1$, $r_1\dot{v}$ and $\dot{z}$ respectively; add and integrate; then, putting $\mu=M_1+M_2=1$,

$$\dot{r}_1^2+r_1^2\dot{v}^2+\dot{z}^2-2\mu/r+\mu/a=2\int\left(\frac{\partial R}{\partial r_1}\cdot\dot{r}_1+\frac{\partial R}{\partial v}\cdot\dot{v}+\frac{\partial R}{\partial z}\cdot\dot{z}\right)dt=2\int d'R, \tag{4}$$

where $\dfrac{d'R}{dt}$ denotes differentiation of R with respect to t only in so far as the time enters through the co-ordinates of the star M_1, and μ/a is the constant of integration.

Again, multiply (1) by r_1, (3) by z and add to (4); then

$$r_1\ddot{r}_1+\dot{r}_1^2+z\ddot{z}+\dot{z}^2=\mu/r-\mu/a+r_1\frac{\partial R}{\partial r_1}+z\frac{\partial R}{\partial z}+2\int d'R,$$

$$i.e.\quad \frac{1}{2}\frac{d^2}{dt^2}(r^2)-\frac{\mu}{r}+\frac{\mu}{a}=r_1\frac{\partial R}{\partial r_1}+z\frac{\partial R}{\partial z}+2\int d'R.$$

Integrating (2) we find, h being a constant of integration,

$$r_1^2\dot{v}=h+\int\frac{\partial R}{\partial v}dt,$$

so that

$$\dot{v} = \frac{h(1+s^2)}{r^2} + \frac{1+s^2}{r^2}\int \frac{\partial R}{\partial v} \cdot dt.$$

We require next the derivatives of $R(r_1, v, z)$ in terms of those of $R(r, v, s)$; it is easily found that

$$r_1\frac{\partial R}{\partial r_1} + z\frac{\partial R}{\partial z} = r\frac{\partial R(r, v, s)}{\partial r} \quad \text{and} \quad \frac{\partial R(r_1, z)}{\partial v} = \frac{\partial R(r, s)}{\partial v}.$$

Hence the equations of motion may be written

$$\frac{1}{2}\frac{d^2}{dt^2}(r^2) - \frac{\mu}{r} + \frac{\mu}{a} = r\frac{\partial R}{\partial r} + 2\int d'R,$$

$$\dot{v} - \frac{h(1+s^2)}{r^2} = \frac{1+s^2}{r^2} \cdot \int \frac{\partial R}{\partial v} dt,$$

$$\ddot{z} + \mu z/r^3 = \frac{\partial R(r, v, z)}{\partial z}.$$

The solution of these equations will introduce 5 more constants, making 7 in all. Since only 6 are independent, we require an additional equation to relate the constants. From (1) and (3) we have

$$r_1\ddot{r}_1 + z\ddot{z} - r_1^2\dot{v}^2 = -\frac{\mu}{r} + r_1\frac{\partial R}{\partial r_1} + z\frac{\partial R}{\partial z} = -\frac{\mu}{r} + \frac{\partial R(r, v, s)}{\partial r}.$$

Now

$$\dot{r}_1^2 + \dot{z}^2 = \dot{r}^2 + r^2\dot{l}^2 \quad \text{where} \quad s = \tan l.$$

$$\therefore\ r_1\ddot{r}_1 + z\ddot{z} = r\ddot{r} + \dot{r}^2 - \dot{r}_1^2 - \dot{z}^2 \quad (\text{since } r_1^2 + z^2 = r^2)$$

$$= r\ddot{r} - r^2\dot{l}^2$$

$$= r\ddot{r} - \frac{r^2\dot{s}^2}{(1+s^2)^2}.$$

$$\therefore\ r\ddot{r} - \frac{r^2\dot{s}^2}{(1+s^2)^2} - \frac{r^2\dot{v}^2}{(1+s^2)} + \frac{\mu}{r} = r\frac{\partial R}{\partial r},$$

or, dividing by r^2,

$$\frac{\ddot{r}}{r} - \frac{\dot{v}^2}{1+s^2} - \frac{\dot{s}^2}{(1+s^2)^2} + \frac{\mu}{r^3} = \frac{1}{r} \cdot \frac{\partial R}{\partial r},$$

and this equation will establish the relation among the constants. These are de Pontécoulant's equations * of motion.

4. *The Development of the Disturbing Function, R.*—The first approximation to the motion of M_1 relative to M_2 is an ellipse and we take for its elements :

a = semi-axis major,
n = the mean motion, with $n^2a^3 = M_1 + M_2$,
e = eccentricity,
i = inclination to plane xy (we write $\tan i = \gamma$),

* See E. W. Brown, *Lunar Theory*, Cambridge, 1896.

$\epsilon - \varpi =$ mean anomaly at time $t = 0$,
$\varpi =$ longitude of periastron,
$\theta =$ longitude of node.

Then the mean anomaly is

$$nt + \epsilon - \varpi$$

and the mean argument of latitude

$$nt + \epsilon - \theta.$$

Now the principal disturbing effects are found to be the motions of the apse and node and take the form of an approximately uniform advance in the case of the apse and approximately uniform retrogression in the case of the node. The motion of M_1 with respect to M_2 will thus be better represented by a moving ellipse whose node and apse lines have arbitrary motions the actual values of which will be determined later. For this purpose, the mean anomaly ϕ is written $\phi = cnt + \epsilon - \varpi$, and the mean argument η of latitude is written $\eta = gnt + \epsilon - \theta$, where c and g determine the motions of the apse and node.

Denote the elliptic elements of the orbit of M with respect to G by accented letters, a', e', n', ϵ' and ϖ'. The mean anomaly in this orbit is

$$\phi' = n't + \epsilon' - \varpi' \quad \text{and} \quad n'^2 a'^3 = M + M_1 + M_2.$$

Write $\xi = (n - n')t + \epsilon - \epsilon'$ and $m = n'/n$. Then the force-function for the motion of M_1 is found to be

$$\begin{aligned}
R = \frac{M(M_1+M_2)}{M+M_1+M_2} \cdot \frac{m^2}{a}\Big[& \tfrac{1}{4} + \tfrac{3}{4}\cos 2\xi - \tfrac{1}{2}e\cos\phi - \tfrac{9}{4}e\cos(2\xi-\phi) \\
& + \tfrac{3}{4}e\cos(2\xi+\phi) + \tfrac{3}{4}e'\cos\phi' + \tfrac{21}{8}e'\cos(2\xi-\phi') - \tfrac{3}{8}e'\cos(2\xi'+\phi') \\
& + \tfrac{3}{8}\cdot\frac{M_2-M_1}{M_2+M_1}\cdot\frac{a}{a'}\cos\xi + \tfrac{5}{8}\cdot\frac{M_2-M_1}{M_2+M_1}\cdot\frac{a}{a'}\cos 3\xi \\
& - \tfrac{3}{8}\gamma^2 - \tfrac{3}{8}\gamma^2\cos 2\xi + \tfrac{3}{8}\gamma^2\cos 2\eta + \tfrac{3}{8}\gamma^2\cos(2\xi-2\eta) + \tfrac{3}{8}e^2 - \tfrac{1}{8}e^2\cos 2\phi \\
& - \tfrac{15}{8}e^2\cos 2\xi + \tfrac{3}{8}e'^2 + \tfrac{9}{8}e'^2\cos 2\phi' + \tfrac{51}{8}e'^2\cos(2\xi-2\phi') - \tfrac{15}{8}e'^2\cos 2\xi \\
& - \tfrac{3}{4}ee'\cos(\phi+\phi') - \tfrac{3}{4}ee'\cos(\phi-\phi') - \tfrac{3}{8}ee'\cos(2\xi+\phi+\phi') \\
& \qquad + \tfrac{21}{8}ee'\cos(2\xi+\phi-\phi') \\
& + \tfrac{9}{8}ee'\cos(2\xi-\phi+\phi') + \tfrac{63}{8}ee'\cos(2\xi-\phi-\phi') \\
& \qquad - \tfrac{3}{16}\cdot\frac{M_2-M_1}{M_2+M_1}\cdot\frac{a}{a'}\cdot e\cos(\xi+\phi) \\
& - \tfrac{15}{16}\cdot\frac{M_2-M_1}{M_2+M_1}\cdot\frac{a}{a'}\cdot e\cos(\xi-\phi) + \tfrac{3}{8}\cdot\frac{M_2-M_1}{M_2+M_1}\cdot\frac{a}{a'}e'\cos(\xi+\phi') \\
& \qquad + \tfrac{9}{8}\cdot\frac{M_2-M_1}{M_2+M_1}\cdot\frac{a}{a'}\cdot e'\cos(\xi-\phi')
\end{aligned}$$

$$+\tfrac{15}{16}\cdot\frac{M_2-M_1}{M_2+M_1}\cdot\frac{a}{a'}\cdot e\cos(3\xi+\phi)-\tfrac{45}{16}\cdot\frac{M_2-M_1}{M_2+M_1}\cdot\frac{a}{a'}\cdot e\cos(3\xi-\phi)$$
$$-\tfrac{5}{8}\cdot\frac{M_2-M_1}{M_2+M_1}\cdot\frac{a}{a'}e'\cos(3\xi+\phi')$$
$$+\tfrac{25}{8}\cdot\frac{M_2-M_1}{M_2+M_1}\cdot\frac{a}{a'}\cdot e'\cos(3\xi-\phi')+\tfrac{9}{64}\cdot\frac{M_1^2-M_1M_2+M_2^2}{(M_1+M_2)^2}\cdot\left(\frac{a}{a'}\right)^2$$
$$+\tfrac{5}{16}\cdot\frac{M_1^2-M_1M_2+M_2^2}{(M_1+M_2)^2}\cdot\left(\frac{a}{a'}\right)^2\cos 2\xi$$
$$+\tfrac{35}{64}\cdot\frac{M_1^2-M_1M_2+M_2^2}{(M_1+M_2)^2}\cdot\left(\frac{a}{a'}\right)^2\cos 4\xi\Big].$$

5. *The Solution of the Equations.*—With the chosen units of mass $\mu=M_1+M_2=1$. Write $\lambda=M/M+M_1+M_2$ and $\beta=M_2-M_1$.

The method of solution is exactly that outlined by E. W. Brown in his description of de Pontécoulant's theory. Briefly, it consists of using the assumed (moving) elliptic orbit to determine the forces leading to the next approximation, then the adjustments due to this more accurate orbit are made and the next higher approximation obtained, and so on. Excepting for the first approximation the expressions occurring are very unwieldy. However, for the considerations of main interest here very few of the higher order terms are of any account.

Now

$$\tfrac{1}{2}\frac{d^2}{dt^2}(r^2)-\frac{1}{r}+\frac{1}{a}=r\frac{\partial R}{\partial r}+2\int d'R.$$

Suppose $1/r=1/r_0+\delta u$, then $r^2=r_0^2-2r_0^3\delta u+3r_0^4\delta u^2$, etc., and hence to the order δu^2 we have

$$\tfrac{1}{2}\frac{d^2}{dt^2}(r_0^2)-\frac{1}{r_0}+\frac{1}{a}-\frac{d^2}{dt^2}(r_0^3\delta u)-\delta u=-\tfrac{3}{2}\frac{d^2}{dt^2}(r_0^2\delta u)^2+P,$$

where

$$P=r\frac{\partial R}{\partial r}+2\int d'R.$$

Now a occurs only in the form

$$r=a\,(1+\text{terms independent of } a),$$

and hence $r\dfrac{\partial R}{\partial r}=a\dfrac{\partial R}{\partial a}$. It follows that for terms in R depending on a^2 we have

$$r\frac{\partial R}{\partial r}=a\frac{\partial R}{\partial a}=2R;$$

for those depending on a^3 we have

$$r\frac{\partial R}{\partial r}=a\frac{\partial R}{\partial a}=3R.$$

Again v occurs only as $v-v'$, and since we have

$$v-v' = \xi + \text{terms independent of } \xi,$$

then

$$\frac{\partial R}{\partial v} = -\frac{\partial R}{\partial v'} = \frac{\partial R}{\partial \xi}.$$

Now $d'R = dR - \frac{\partial R}{\partial v'} \cdot dv' - \frac{\partial R}{\partial r'} \cdot dr'$ by definition, where $r' = R$ the radius vector in the orbit of the star M; so that for the part R_p of R of order r^{p+2}/R^{p+3}, P can be put into either of the forms

$$P = (p+4)R_p + 2(p+3)\int R_p \frac{dr'}{r'} + 2\int \frac{\partial R}{\partial \xi} dv'$$

$$= (p+2)R_p + 2(p+3)\int R_p \frac{dr}{r} + 2\int \frac{\partial R}{\partial \xi} dv.$$

Take $r_0 = a$ as first approximation. Neglect δu^2, then

$$-\frac{1}{n^2} \cdot \frac{d^2}{dt^2}(\delta u) - \delta u = P.$$

We are neglecting e and e' so that $R = \frac{\lambda m^2}{a} \cdot (\frac{1}{4} + \frac{3}{4} \cos 2\xi)$; also $P = 4R + 2\int \frac{\partial R}{\partial \xi} dv'$, and since $v' = n't + \epsilon'$ the equation becomes

$$-\frac{1}{n^2} \cdot \frac{d^2}{dt^2}(\delta u) - \delta u = \frac{\lambda m^2}{a}(1 + 3\cos 2\xi) + \frac{3\lambda m^2}{2a} \cdot \frac{m}{1-m} \cos 2\xi + \frac{\alpha}{a},$$

where α/a is the constant of integration appearing in P. Assuming $a\delta u = b_0 + b_2 \cos 2\xi$, we readily find

$$b_0 = -\lambda m^2 - \alpha,$$
$$b_2 = \lambda m^2 + \tfrac{19}{6}\lambda m^3;$$

as yet b_0 remains undetermined.

The longitude equation is

$$\dot{v} - \frac{h(1+s^2)}{r^2} = \frac{1+s^2}{r^2}\int \frac{\partial R}{\partial v} dt,$$

and at first we neglect the inclination, *i.e.* we neglect s^2. Put $v = v_0 + \delta v$, $h = h_0 + \delta h$ where the suffix o denotes the modified elliptic value. Since s is neglected,

$$h_0 = \sqrt{a(1-e^2)} = na^2, \qquad e^2 \text{ being neglected.}$$

$$\therefore \frac{d}{dt}(\delta v) = \frac{2h_0}{r_0} \cdot \delta u + \frac{\delta h}{{r_0}^2} + \frac{1}{{r_0}^2}\int \frac{\partial R}{\partial v} dt,$$

and, remembering that as a first approximation $r_0 = a$, we obtain

$$\frac{d}{dt}(\delta v) = 2na\delta u + \frac{\delta h}{a^2} + \frac{1}{a^2}\int \frac{\partial R}{\partial \xi} dt$$

$$= 2n\{b_0 + (\lambda m^2 + \tfrac{19}{6}\lambda m^3)\cos 2\xi\} + \tfrac{3}{4}\frac{\lambda m^2 n^2}{n - n'}\cos 2\xi + \frac{\delta h}{a^2},$$

$$\therefore\ \delta v = \left(2nb_0 + \frac{\delta h}{a^2}\right)t + (\tfrac{11}{8}\lambda m^2 + \tfrac{59}{12}\lambda m^3)\sin 2\xi.$$

Now n is defined to be the coefficient of t in the final expression for the longitude so that $2nb_0a^2 + \delta h = 0$ and $\delta v = (\tfrac{11}{8}\lambda m^2 + \tfrac{59}{12}\lambda m^3)\sin 2\xi$ to order m^3.

To determine b_0 we use the equation giving the relation between the arbitrary constants. Neglecting the inclination and a/a' it becomes

$$\ddot{r}/r - \dot{v}^2 + 1/r^3 = 2R/r^2.$$

We substitute in this the values so far obtained for $1/r$ and v and equate to zero the constant term; this gives $b_0 = \dfrac{\lambda m^2}{6}$. Hence for the terms in m only, as far as m^3,

$$a/r = 1 + \tfrac{1}{6}\lambda m^2 + (\lambda m^2 + \tfrac{19}{6}\lambda m^3)\cos 2\xi,$$
$$v = nt + \epsilon + (\tfrac{11}{8}\lambda m^2 + \tfrac{59}{12}\lambda m^3)\sin 2\xi.$$

6. *The Determination of* c *and* g.—Neglecting a/a', e' and γ we have to this order

$$R = \frac{\lambda m^2}{a}\cdot[\tfrac{1}{4} + \tfrac{3}{4}\cos 2\xi - \tfrac{1}{2}e\cos\phi - \tfrac{9}{4}e\cos(2\xi - \phi) + \tfrac{3}{4}e\cos(2\xi + \phi)]$$

and

$$r_0 = a(1 - e\cos\phi), \qquad v_0 = nt + \epsilon + 2e\sin\phi, \qquad r' = a', \qquad v' = n't + \epsilon'.$$

Neglecting m^3 and taking account of terms in e only

$$P = 4R + 2\int \frac{\partial R}{\partial \xi} dv' = 4R + 2n'\int \frac{\partial R}{\partial \xi} dt$$

$$= -\frac{\lambda m^2 e}{a}[2\cos\phi + 9\cos(2\xi - \phi) - 3\cos(2\xi + \phi)].$$

Take for the particular integral

$$a\delta u = \tfrac{1}{6}\lambda m^2 + \lambda m^2\cos 2\xi + ec_0\cos\phi + ec_1\cos(2\xi + \phi) + ec_{-1}\cos(2\xi - \phi).$$

This has to be substituted in the equation

$$\tfrac{1}{2}\frac{d^2}{dt^2}(r_0^2) - \frac{1}{r_0} + \frac{1}{a} - \frac{d^2}{dt^2}(r_0^3\delta u) - \delta u = -\tfrac{3}{2}\frac{d^2}{dt^2}(r_0^2\delta u)^2 + P.$$

The coefficients of $\cos\phi$, $\cos(2\xi - \phi)$ and $\cos(2\xi + \phi)$ give

$$c^2 - 1 + c^2(c_0 - \tfrac{1}{2}\lambda m^2) - c_0 = -2\lambda m^2$$
$$(2 - 2m - c)^2(c_{-1} - \tfrac{3}{2}\lambda m^2) - c_{-1} = -9\lambda m^2$$
$$(2 - 2m + c)^2(c_1 - \tfrac{3}{2}\lambda m^2) - c_1 = 3\lambda m^2.$$

Now c_0, c_1 and c_{-1} are of order m at most, so that from the first of these, since $(c^2-1)(1+c_0) = -2\lambda m^2 + \frac{1}{2}\lambda m^2 c^2$, we see that c^2-1 is of order m^2 and hence

$$c = 1 - \tfrac{3}{4}\lambda m^2.$$

A more accurate investigation gives

$$c = 1 - \tfrac{3}{4}\lambda m^2 - \tfrac{225}{32}\lambda^2 m^3,$$

and for the motion of the node it is found in a similar way that

$$g = 1 + \tfrac{3}{4}\lambda m^2 - \tfrac{9}{32}\lambda^2 m^3.$$

These are the parts of c and g which depend on m only and are sufficiently accurate for the present purpose.

The remaining equations give

$$c_1 = \tfrac{33}{16}\lambda m^2 \quad \text{and} \quad c_{-1} = \tfrac{15}{8}\lambda m.$$

We leave c_0 arbitrary until the longitude terms are found.

To the order $m^2 e$ we have

$$\frac{dv_0}{dt} - \frac{h_0}{{r_0}^2} + \frac{d}{dt}(\delta v) = \frac{2h_0}{r_0}\,.\,\delta u + \frac{\delta h}{{r_0}^2} + \frac{1}{{r_0}^2}\int \frac{\partial R}{\partial \xi}\,dt.$$

Omitting terms independent of e we find that

$$\frac{1}{n}\frac{d}{dt}(\delta v) = \tfrac{1}{3}\lambda m^2 + na\delta h + e(-2c + 2 + 2c_0 + \tfrac{1}{3}\lambda m^2 + 2na\delta h)\cos\phi + \tfrac{15}{4}\lambda m e\cos(2\xi - \phi) + \tfrac{51}{8}\lambda m^2 e\cos(2\xi + \phi).$$

The constant term being zero gives $\delta h = -\frac{1}{3}\lambda m^2 na^2$. We choose c_0 so that the coefficient of $\sin\phi$ in the disturbed motion is the same as in elliptic motion; thus the coefficient of $\cos\phi$ in the above is zero, *i.e.* $c_0 = -\frac{7}{12}\lambda m^2$. Then $\delta v = \frac{15}{4}\lambda m e \sin(2\xi - \phi) + \frac{17}{8}\lambda m^2 e \sin(2\xi + \phi)$.

The terms in e', a/a' and γ are found in a similar way. In obtaining the higher approximations care must be taken to include the changes in R due to the lower approximations. The work is very lengthy even when terms in m^3 are neglected.

7. *The Values of the Co-ordinates.*—The following are the results obtained up to terms of such an order as the present determinations of the elements and dimensions of the Algol system indicate as being desirable (McLaughlin, *loc. cit.*):—

The few terms of the next higher order for the particular values under consideration are also given.

$$a/r = 1 + (e - \tfrac{1}{8}e^3)\cos\phi + e^2\cos 2\phi + \tfrac{9}{8}e^3\cos 3\phi - \tfrac{5}{8}e\gamma^2\cos(\phi - 2\eta) + \tfrac{15}{8}\lambda m e\cos(2\xi - \phi) - \tfrac{15}{16}\lambda m\beta\frac{a}{a'}\cos\xi + \tfrac{5}{4}\beta\frac{a}{a'}e'\cos(\xi + \phi'),$$

together with the following higher order terms:—

$$+\tfrac{1}{6}\lambda m^2-\tfrac{1}{3}e^4\cos 2\phi+\tfrac{4}{3}e^4\cos 4\phi+\tfrac{21}{8}\lambda mee'\cos(\phi-\phi'),$$
$$-\tfrac{21}{8}\lambda mee'\cos(\phi+\phi')-\tfrac{5}{4}e^2\gamma^2\cos 2\eta+(\lambda m^2+\tfrac{15}{4}\lambda me^2)\cos 2\xi,$$
$$-\tfrac{15}{8}\lambda mee'\cos(2\xi-\phi+\phi')+\tfrac{35}{8}\lambda mee'\cos(2\xi-\phi-\phi'),$$
$$+\tfrac{5}{2}\lambda\beta\frac{a}{a'}ee'\cos(\xi+\phi+\phi').$$

$$v=nt+\epsilon+(2e-\tfrac{1}{4}e^3)\sin\phi+\tfrac{5}{4}e^2\sin 2\phi+\tfrac{13}{12}e^3\sin 3\phi,$$
$$-3\lambda me'\sin\phi'-\tfrac{9}{4}\lambda me'^2\sin 2\phi'-\tfrac{1}{4}\gamma^2\sin 2\eta,$$
$$+\tfrac{3}{4}e\gamma^2\sin(\phi-2\eta)-\tfrac{1}{2}e\gamma^2\sin(\phi+2\eta)+\tfrac{15}{4}\lambda me\sin(2\xi-\phi),$$
$$-\tfrac{15}{8}\lambda m\beta\frac{a}{a'}\sin\xi+\tfrac{5}{2}\beta\frac{a}{a'}e'\sin(\xi+\phi'),$$

together with the following higher order terms:—

$$-(\tfrac{11}{24}e^4+\tfrac{5}{16}e^2\gamma^2)\sin 2\phi+\tfrac{103}{96}e^4\sin 4\phi-\tfrac{27}{8}\lambda me'^3\sin\phi',$$
$$-\tfrac{53}{24}\lambda me'^3\sin 3\phi'+\tfrac{21}{4}\lambda mee'\sin(\phi-\phi')-\tfrac{21}{4}\lambda mee'\sin(\phi+\phi'),$$
$$-(\tfrac{9}{16}e^2\gamma^2+\tfrac{1}{4}\gamma^4)\sin 2\eta-\tfrac{1}{8}e^2\gamma^2\sin(2\phi-2\eta)-\tfrac{13}{16}e^2\gamma^2\sin(2\phi+2\eta),$$
$$+\tfrac{1}{32}\gamma^4\sin 4\eta+(\tfrac{11}{8}\lambda m^2+\tfrac{75}{16}\lambda me^2-\tfrac{3}{16}\lambda m\gamma^2)\sin 2\xi,$$
$$+\tfrac{45}{16}\lambda me^2\sin(2\xi-2\phi)-\tfrac{15}{4}\lambda mee'\sin(2\xi-\phi+\phi'),$$
$$+\tfrac{35}{4}\lambda mee'\sin(2\xi-\phi-\phi')+\tfrac{9}{16}\lambda m\gamma^2\sin(2\xi-2\eta).$$

$$l=(1-e^2-\tfrac{3}{8}\gamma^2)\gamma\sin\eta+e\gamma\sin(\phi-\eta)+e\gamma\sin(\phi+\eta),$$
$$+\tfrac{3}{4}e^2\gamma\sin(2\phi-\eta)+\tfrac{9}{8}e^2\gamma\sin(2\phi+\eta)-\tfrac{1}{24}\gamma^3\sin 3\eta,$$
$$+\tfrac{3}{8}\lambda m\gamma\sin(2\xi-\eta)+\tfrac{5}{4}\beta\frac{a}{a'}e'\gamma\sin(\xi+\phi'-\eta)+\tfrac{5}{4}\beta\frac{a}{a'}e'\gamma\sin(\xi+\phi'+\eta),$$

together with the following higher order terms:—

$$-(\tfrac{5}{8}e^3\gamma-\tfrac{3}{4}e\gamma^3)\sin(\phi-\eta)-(\tfrac{5}{4}e^3\gamma-\tfrac{1}{8}e\gamma^3)\sin(\phi+\eta),$$
$$-\tfrac{3}{8}\lambda me'\gamma\sin(\phi'-\eta)-\tfrac{3}{8}\lambda me'\gamma\sin(\phi'+\eta)+\tfrac{17}{24}e^3\gamma\sin(3\phi-\eta),$$
$$+\tfrac{4}{3}e^3\gamma\sin(3\phi+\eta)+\tfrac{1}{2}e\gamma^3\sin(\phi-3\eta)-\tfrac{1}{8}e\gamma^3\sin(\phi+3\eta),$$
$$+\tfrac{15}{8}\lambda me\gamma\sin(2\xi-\phi+\eta)+\tfrac{3}{2}\lambda me\gamma\sin(2\xi-\phi-\eta),$$
$$+\tfrac{3}{8}\lambda me\gamma\sin(2\xi+\phi-\eta)+\tfrac{7}{8}\lambda me'\gamma\sin(2\xi-\phi'-\eta),$$
$$-\tfrac{3}{8}\lambda me'\gamma\sin(2\xi+\phi'-\eta),$$

where

$$\phi=cnt+\epsilon-\varpi,\qquad \eta=gnt+\epsilon-\theta,$$
$$\xi=(n-n')t+\epsilon-\epsilon'\quad\text{and}\quad\phi'=n't+\epsilon'-\varpi';$$

and

$$\lambda=M/(M+M_1+M_2)\quad\text{and}\quad\beta=(M_2-M_1)/(M_2+M_1).$$

8. *Co-ordinates Referred to the Centre of Mass of the Whole System.*— The expressions just given are the co-ordinates of the star M_1 with respect to axes in fixed directions and through M_2. Now the centre of mass of M_1 and M_2 has an elliptic motion about the centre of mass of the whole system. Also it is the resultant of this motion and the motion of M_1 about the centre of mass of M_1 and M_2 that can be observed, so that a formula for the distance of M_1 from any plane through the centre of mass of the whole system will give the co-ordinates of M_1 with respect to unaccelerated axes and a means of comparing the theory with observations.

If the direction cosines (with respect to xyz) of the normal to this plane are $\cos A \cos B$, $\sin A \cos B$ and $\sin B$ (*i.e.* the normal is towards a point of longitude A and latitude B), then the distance of M_1 from this plane is given by

$$\rho(A, B) = m_2 r \cos l \cos (v - A) \cos B - \lambda R \cos (v' - A) \cos B + m_2 r \sin l \sin B$$

where

$$m_2 = \frac{M_2}{M_1 + M_2}.$$

If we write $\bar{\phi} = nt + \epsilon - A$ and $\bar{\phi}' = n't + \epsilon' - A$, then

$$\begin{aligned}\rho(A, B) = m_2 a \cos B[&(1 - \tfrac{1}{2}e^2 - \tfrac{1}{4}\gamma^2) \cos \bar{\phi} + (\tfrac{1}{2}e - \tfrac{3}{8}e^3 - \tfrac{1}{8}e\gamma^2) \cos (\phi + \bar{\phi})\\ &- (\tfrac{3}{2}e - \tfrac{3}{8}e\gamma^2) \cos (\phi - \bar{\phi}) + \tfrac{1}{8}e^2 \cos (2\phi - \bar{\phi}) + \tfrac{3}{8}e^2 \cos (2\phi + \bar{\phi})\\ &+ \tfrac{1}{24}e^3 \cos (3\phi - \bar{\phi}) + \tfrac{1}{8}e^3 \cos (3\phi + \bar{\phi}) + \tfrac{1}{4}\gamma^2 \cos (\bar{\phi} - 2\eta)\\ &- \tfrac{1}{2}e\gamma^2 \cos (\phi - \bar{\phi} - 2\eta) + \tfrac{3}{8}e\gamma^2 \cos (\phi + \bar{\phi} - 2\eta)\\ &+ \tfrac{1}{8}e\gamma^2 \cos (\phi - \bar{\phi} + 2\eta)\\ &+ \tfrac{3}{2}\lambda m e' \cos (\bar{\phi} - \phi') - \tfrac{3}{2}\lambda m e' \cos (\bar{\phi} + \phi') + \tfrac{15}{16}\lambda m e \cos (2\xi - \phi + \bar{\phi})\\ &- \tfrac{45}{16}\lambda m e \cos (2\xi - \phi - \bar{\phi}) + \tfrac{45}{32}\lambda m \beta \frac{a}{a'} \cos (\xi - \bar{\phi})\\ &- \tfrac{15}{32}\lambda m \beta \frac{a}{a'} \cos (\xi + \bar{\phi}) - \tfrac{15}{8}\beta \frac{a}{a'} e' \cos (\xi + \phi' - \bar{\phi})\\ &+ \tfrac{5}{8}\beta \frac{a}{a'} e' \cos (\xi + \phi' + \bar{\phi})]\end{aligned}$$

$$\begin{aligned}&- \lambda a' \cos B[(1 - \tfrac{1}{2}e'^2) \cos \bar{\phi}' + (\tfrac{1}{2}e' - \tfrac{3}{8}e'^3) \cos (\phi' + \bar{\phi}')\\ &\quad - \tfrac{3}{2}e' \cos (A - \varpi') + \tfrac{3}{8}e'^2 \cos (2\phi' + \bar{\phi}') + \tfrac{1}{8}e'^2 \cos (2\phi' - \bar{\phi}')\\ &\quad + \tfrac{1}{3}e'^3 \cos (3\phi' + \bar{\phi}') + \tfrac{1}{24}e'^3 \cos (3\phi' - \bar{\phi}')]\\ &+ m_2 a \sin B[(\gamma - \tfrac{1}{2}e\gamma^2 - \tfrac{5}{8}\gamma^3) \sin \eta + \tfrac{3}{2}e\gamma \cos (\phi - \eta)\\ &\quad + \tfrac{1}{2}e\gamma \sin (\phi + \eta) + \tfrac{1}{2}e^2\gamma \sin (2\phi - \eta) + \tfrac{3}{8}e^2\gamma \sin (2\phi + \eta)\\ &\quad + \tfrac{1}{24}\gamma^3 \sin 3\eta + \tfrac{3}{8}\lambda m \gamma \sin (2\xi - \eta) + \tfrac{5}{4}\beta \frac{a}{a'} \gamma \sin (\xi + \phi' - \eta)].\end{aligned}$$

The velocity of M_1 in direction (A, B) is simply $\dfrac{d\rho(A, B)}{dt}$. The rectangular co-ordinates of M_1 are $\rho(0, 0)$, $\rho(\pi/2, 0)$ and $\rho(-, \pi/2)$.

9. *The Radial Velocity of the Star M_1.*—If the direction (A, B) is taken to be that of the observer, then the rate of approach of M_1 is $-\dfrac{d\rho(A, B)}{dt}$, and this can be obtained at once from the series given for $\rho(A, B)$. We find

$$\begin{aligned}-\frac{d\rho(A, B)}{dt} = m_2 a n \cos B[&(1 - \tfrac{1}{2}e^2 - \tfrac{1}{4}\gamma^2) \sin \bar{\phi} + (e - \tfrac{3}{4}e^3 - \tfrac{1}{4}e\gamma^2) \sin (\phi + \bar{\phi})\\ &+ \tfrac{1}{8}e^2 \sin (2\phi - \bar{\phi}) + \tfrac{9}{8}e^2 \sin (2\phi + \bar{\phi}) + \tfrac{1}{12}e^3 \sin (3\phi - \bar{\phi})\\ &+ \tfrac{4}{3}e^3 \sin (3\phi + \bar{\phi}) - \tfrac{1}{4}\gamma^2 \sin (\phi - 2\eta) + e\gamma^2 \sin (\phi - \bar{\phi} - 2\eta)\\ &+ \tfrac{1}{4}e\gamma^2 \sin (\phi - \bar{\phi} + 2\eta) + \tfrac{3}{2}\lambda m e' \sin (\bar{\phi} - \phi')\\ &- \tfrac{3}{2}\lambda m e' \sin (\bar{\phi} + \phi') + \tfrac{15}{8}\lambda m e \sin (2\xi - \phi + \bar{\phi})\\ &- \tfrac{15}{16}\lambda m \beta \frac{a}{a'} \sin (\xi + \bar{\phi}) + \tfrac{5}{4}\beta \frac{a}{a'} e' \sin (\xi + \phi' + \bar{\phi})]\end{aligned}$$

$$-\lambda a'n' \cos B[(1-\tfrac{1}{2}e'^2)\sin\bar{\phi}'+(e'-\tfrac{3}{4}e'^3)\sin(\phi'+\bar{\phi}')$$
$$+\tfrac{1}{8}e'^2\sin(2\phi'-\bar{\phi}')+\tfrac{9}{8}e'^2\sin(2\phi'+\bar{\phi}')$$
$$+\tfrac{13}{12}e'^3\sin(3\phi'-\bar{\phi}')+\tfrac{4}{3}e'^3\sin(3\phi'+\bar{\phi}')]$$
$$-m_2an\sin B[(\gamma-\tfrac{1}{2}e^2\gamma-\tfrac{5}{8}\gamma^3)\cos\eta+e\gamma\cos(\phi+\eta)$$
$$+\tfrac{1}{2}e^2\gamma\cos(2\phi-\eta)+\tfrac{9}{8}e^2\gamma\cos(2\phi+\eta)$$
$$+\tfrac{1}{8}\gamma^3\cos 3\eta+\tfrac{3}{8}\lambda m\gamma\cos(2\xi-\eta)].$$

The chief terms are those in $\sin\bar{\phi}$, $\sin(\phi+\bar{\phi})$ and those in the second square brackets due to the motion about the centre of mass of the whole system. The terms due to the perturbation of the orbit of M_1 relative to M_2 are seen to be very small indeed.

10. *The Period as an Orbital Element.*—In an undisturbed elliptic orbit the period of the light curve is equal to the period in the ellipse. We proceed to find to what extent this is modified when the orbit is disturbed by a third star.

Now in § 3, equation (4), we had, V being the velocity,

$$V^2-2\mu/r+\mu/a=2\int d'R.$$

Also in elliptic motion if V is the velocity, μ the sum of the masses, r the radial distance and α the semi-axis major,

$$V^2-2\mu/r+\mu/\alpha=0,$$

so that the axis of the instantaneous orbit is given by

$$1/\alpha=1/a-2\int d'R.$$

We easily find that as far as terms in $m^2e'^2$

$$\int d'R=R-\frac{\lambda m^2}{a}\,.\,[\tfrac{3}{4}e'\cos\phi'+\tfrac{9}{8}e'^2\cos 2\phi'],$$

and hence to this order,

$$1/\alpha=1/a-2R+\frac{\lambda m^2}{a}\,.\,[\tfrac{3}{2}e'\cos\phi'+\tfrac{9}{4}e'^2\cos 2\phi'].$$

If $\nu^2\alpha^3=1$, then $2\pi/\nu$ is the required period, and from the above and the expression for R we get

$$2\pi/\nu=2\pi/n[1+\tfrac{3}{4}\lambda m^2+\tfrac{9}{4}\lambda m^2\cos 2\xi-\tfrac{3}{2}\lambda m^2e\cos\phi$$
$$-\tfrac{27}{4}\lambda m^2e\cos(2\xi-\phi)+\tfrac{9}{4}\lambda m^2e\cos(2\xi+\phi)$$
$$+\tfrac{63}{8}\lambda m^2e'\cos(2\xi-\phi')-\tfrac{9}{8}\lambda m^2e'\cos(2\xi+\phi')$$
$$+\tfrac{9}{8}\lambda m^2\beta\frac{a}{a'}\cos\xi+\tfrac{135}{8}\lambda m^2\beta\frac{a}{a'}\cos 3\xi$$
$$+\text{higher order terms}],$$

and it will be noticed that there are no long-period terms with coefficients of any appreciable magnitude.

If it were possible to study the orbital period in the same way as the period of the light curve, the absence of long-period terms would cause the orbital period to appear constant and we should find a difference in the two periods expressed by $\frac{2\pi}{\nu} = \frac{2\pi}{n}(1 + \frac{3}{4}\lambda m^2)$; this gives the forward motion of the apse.

11. *The Change in Period as obtained from the Light Curve.*—If the orbit of M_1 relative to M_2 were plane, successive minima would occur at intervals during which v increased by 2π. Hence, whatever the true criterion for a minimum in the theoretical light curve it must reduce to this when the inclination is neglected. [If the observer is in a direction of longitude v_0 and latitude l_0 then successive eclipses occur at times given by the solution of

$$\sin l \tan l_0 + \cos l \cos (v - v_0) = \text{const.}$$

regarded as an equation in t.] Hence the criterion for the increase of v by 2π will differ from the true criterion only in the terms in γ, and, for simplicity, we will investigate the changes in period due to the remaining terms.

To obtain the instants at which v passes through a certain value, we must first invert the series for v. Omitting the terms in γ, we find

$$\begin{aligned} nt = \; & v - \epsilon + (2e - \tfrac{5}{2}e^3)\sin(cv - c\epsilon + \epsilon - \varpi) \\ & + \tfrac{13}{4}e^2 \sin 2(cv - c\epsilon + \epsilon - \varpi) + \tfrac{47}{6}e^3 \sin 3(cv - c\epsilon + \epsilon - \varpi) \\ & - 3\lambda me' \sin(mv - m\epsilon + \epsilon' - \varpi') - \tfrac{9}{4}\lambda me'^2 \sin 2(mv - m\epsilon + \epsilon' - \varpi') \\ & - \tfrac{53}{24}\lambda me'^3 \sin 3(mv - m\epsilon + \epsilon' - \varpi') \\ & + \tfrac{21}{4}\lambda mee' \sin\{(c-m)v - (c-m-1)\epsilon - \epsilon' - \varpi + \varpi'\} \\ & - \tfrac{21}{4}\lambda mee' \sin\{(c+m)v - (c+m-1)\epsilon + \epsilon' - \varpi - \varpi'\} \\ & + \tfrac{15}{4}\lambda me \sin\{(2-2m-c)v + (c+2m-1)\epsilon - 2\epsilon' + \varpi\} \\ & + \tfrac{35}{4}\lambda mee' \sin\{(2-3m-c)v + (c+3m-1)\epsilon - 3\epsilon' + \varpi + \varpi'\} \\ & - \tfrac{15}{4}\lambda mee' \sin\{(2-m-c)v + (c+m-1)\epsilon - \epsilon' + \varpi - \varpi'\} \\ & - \tfrac{15}{8}\lambda m\beta\frac{a}{a'} \sin\{(1-m)v + (2-m)\epsilon - \epsilon'\} \\ & + \tfrac{5}{2}\beta\frac{a}{a'}e' \sin\{v - \varpi'\}. \end{aligned}$$

Now it must be remembered that the light of a star is seen a considerable time after it is emitted, the actual interval being of course proportional to the distance of the star. The distance of the star M_1 from the "fixed" centre of mass of the whole system has been shown to vary, and it follows therefore that the time difference between the actual occurrence of a minimum and an observation of it will vary in precisely the same way.

Consider an eclipse of M_1; it occurs at a calculated time t. The distance of M_1 from the observer differs from ρ only by a constant, and hence the observer sees the event at time $t + \rho/c$, where c is the velocity of light, and this must be expressed in terms of v, the longitude of M_1.

We already have t in terms of v and ρ in terms of t so we can immediately

find ρ in terms of v; we then derive the time T, say, at which the star M_1 is seen when it is at longitude v. We find

$$\begin{aligned}
T &= \text{const.} + t + \rho/c \\
&= \text{const.} + \frac{v}{n} + \frac{2e}{n}\sin(cv - c\epsilon + \epsilon - \varpi) \\
&\quad - 3\lambda m\frac{e'}{n}\sin(mv - m\epsilon + \epsilon' - \varpi') - \tfrac{9}{4}\lambda m\frac{e'^2}{n}\sin 2(mv - m\epsilon + \epsilon' - \varpi') \\
&\quad + m_2 \,.\, a/c\cos(v - A) \\
&\quad - \lambda \,.\, a'/c\{(1 - \tfrac{1}{2}e'^2)\cos(mv - m\epsilon + \epsilon' - A) \\
&\qquad - (\tfrac{1}{2}e' - \tfrac{3}{8}e'^3)\cos(2mv - 2m\epsilon + 2\epsilon' - A - \varpi') \\
&\qquad - \tfrac{3}{8}e'^2\cos(3mv - 3m\epsilon + 3\epsilon' - 2\varpi' - A) \\
&\qquad - \tfrac{1}{8}e'^2\cos(mv - m\epsilon + \epsilon' - 2\varpi' + A) \\
&\qquad - \tfrac{1}{3}e'^3\cos(4mv - 4m\epsilon + 4\epsilon' - 3\varpi' - A) \\
&\qquad - \tfrac{1}{24}e'^3\cos(2mv - 2m\epsilon + 2\epsilon' - 3\varpi' + A)\},
\end{aligned}$$

together with some smaller terms.

Suppose now we measure the time at which M_1 is in a certain longitude v, say, at which a minimum occurs. The time T_r at which the rth next minimum occurs is then given by

$$\begin{aligned}
T_r &= \text{const.} + \frac{2\pi}{n} \,.\, r + \frac{2e}{n}\sin(\tfrac{3}{2}\pi\lambda m^2 r + \text{const.}) \\
&\quad - 3\lambda m\frac{e'}{n}\sin(2\pi mr + \text{const.}) - \tfrac{9}{4}\lambda m e'^2/n\sin(4\pi mr + \text{const.}) \\
&\quad - \lambda(1 - \tfrac{1}{2}e'^2)\frac{a'}{c}\cos(2\pi mr + \text{const.}) \\
&\quad + \tfrac{1}{2}\lambda e'\frac{a'}{c}\cos(4\pi mr + \text{const.}) + \text{smaller terms.}
\end{aligned}$$

We have thus obtained a formula similar in type to the empirical formula of Hellerich. It remains to investigate the amplitudes and periods of the various terms.

In the case of Algol, the mean period of the close pair $\frac{2\pi}{n}$ is 2·86731 days and the ratio of the periods is $m = 0{\cdot}00416$. Also, in terms of the units of mass used, $M_1 = \frac{5}{6}$, $M_2 = \frac{1}{6}$ and $M = 0{\cdot}41$; these give $\beta = -\frac{2}{3}$ and $\lambda = 0{\cdot}29$. McLaughlin's more recent determinations give $M_1 = \frac{4}{5}$ and $M_2 = \frac{1}{5}$.

Originally, McLaughlin gave the eccentricity of the short-period orbit as 0·13, but the new determination is 0·033. The value of the long-period eccentricity e' is 0·26. The value given for the major-axis of the long-period orbit is of the order of 300 c where c is the distance travelled by light in 1 sec.; and accordingly a is of order 10 c.

Using these values of the constants, we find

$$\frac{2e}{n} = \frac{2\pi}{n} \cdot \frac{e}{\pi} = 0^d\cdot 118, \text{ if } e = 0\cdot 13$$

and $0^d\cdot 03$ if $e = 0\cdot 033$

$$3\lambda m'/n = 0^d\cdot 00043 = 37 \text{ secs.}$$
$$\lambda(1 - \tfrac{1}{2}e'^2)a'/c = 90 \text{ secs.}$$
$$\tfrac{1}{2}\lambda e'a'/c = 4 \text{ secs.}$$
$$\tfrac{9}{4}\lambda m'^2/n = 7 \text{ secs.}$$

and

$$\tfrac{3}{2}\pi\lambda m^2 = 0^\circ\cdot 0014, \qquad 2\pi m = 1^\circ\cdot 5.$$

Inserting these values, the formula becomes

$$T_r = \text{const.} + 2^d\cdot 86731r + 0^d\cdot 118 \sin\{0^\circ\cdot 0014(r - \text{const.})\} - 37^{\text{secs}} \sin\{1^\circ\cdot 5(r - \text{const.})\} - 90^{\text{secs}} \sin\{1^\circ\cdot 5(r - \text{const.})\} - 7^{\text{secs}} \sin\{3^\circ(r - \text{const.})\} + 4^{\text{secs}} \sin\{3^\circ(r - \text{const.})\},$$

the constants involved all being different of course.

The terms of the same period combine to give a single term of the same period and an amplitude of the same order as the original amplitudes. T_r is therefore of the form

$$T_r = \text{const.} + 2^d\cdot 86731r + 0^d\cdot 118 \sin\{0^\circ\cdot 0014(r - \text{const.})\} + 60^{\text{secs}} \sin\{1^\circ\cdot 5(r - \text{const.})\} + 5^{\text{secs}} \sin\{3^\circ(r - \text{const.})\}.$$

The period of the first term is about 1000 years, while that of the second periodic term is just under 1 year.

Finally we will find the time between the rth and $(r+1)$th minima and so determine how the theoretical period varies. We have

$$T_{r+1} - T_r = 2^d\cdot 86731 + 0^d\cdot 118[\sin\{0^\circ\cdot 0014(r + 1 - \text{const.}) - \sin[0^\circ\cdot 0014(r - \text{const.})]\}] + 60^{\text{secs}}[\sin\{1^\circ\cdot 5(r + 1 - \text{const.}) - \sin[1^\circ\cdot 5(r - \text{const.})]\}]$$
$$= 2^d\cdot 86731 + 0^s\cdot 25 \sin\{0^\circ\cdot 0014(r - \text{const.})\} + 1^s\cdot 6 \sin\{1^\circ\cdot 5(r - \text{const.})\}.$$

It follows that the period determined in this way, *i.e.* from observations of two consecutive minima, can vary during a comparatively short time (two years) by as much as three seconds, approximately.

Clare College, Cambridge:
1934 *September* 29.

THE PROPER MOTIONS OF THE SHORT-PERIOD CEPHEIDS.

Alan Fletcher, M.A., Ph.D.

An important addition to our knowledge of the proper motions of the short-period or cluster-type Cepheids has recently been made by Mrs. P. Fairfield Bok and Miss C. D. Boyd.* They have derived proper motions for 43 of these stars from Harvard plates, and by adding material from other sources have made a list of the proper motions of 58 stars. Using the solar apex and velocity ($U = 109$ km./sec.) found by Strömberg from 26 radial velocities, and roughly checking the apex by the 58 proper motions, they find from the parallactic motion an absolute magnitude $+0^m.05 \pm 0^m.18$. Making from 22 radial velocities their own estimate of the mean residual ($\overline{V'} = 77$ km./sec.), they find $+0^m.16 \pm 0^m.31$ from the peculiar motions (other than the v-residuals, which they do not consider). The final mean, $+0^m.08 \pm 0^m.15$, implies a correction of about the same amount to Shapley's revised zero-point, which gives $0^m.00$ as the absolute magnitude of most of the short-period Cepheids. These determinations depend upon comparisons of proper motions with radial velocities. Fresh radial-velocity material is on the way,† and in this paper we do not investigate the values of U and $\overline{V'}$. We consider only the proper motions.

The range in absolute magnitude is probably small, and Mrs. Bok and Miss Boyd have reduced all proper motions to the eleventh apparent magnitude. The apparent magnitudes range from 7·0 to 12·2, and the measured motions have fairly uniform probable errors; thus the reduced motions have very diverse observational errors, together with fairly uniform "cosmical errors." Since the question of weights would be less important if bright and faint stars were to agree, let us roughly examine the motions for various magnitude-groups. The columns of Table I give for each group the range in magnitude, the number of stars, the parallactic motion q derived from the unweighted reduced motions by the formula ‡ $q = -\Sigma(v \sin \lambda)/\Sigma(\sin^2 \lambda)$ and the corresponding absolute magnitude, assuming $U = 109$ km./sec.

TABLE I

m	n	q	M
		″	m
7·0–10·0	19	0·0238	+1·07
10·2–11·2	20	0·0033	−3·24
11·5–12·2	19	0·0222	+0·92
7·0–10·4	29	0·0174	+0·40
10·7–12·2	29	0·0139	−0·09
7·0–12·2	58	0·0155	+0·15

* *H.B.*, 893, 1933.

† Joy, *P.A.S.P.*, 44, 240, 1932.

‡ To avoid confusion, we follow Mrs. Bok and Miss Boyd in measuring v positive towards the apex.

The violent oscillation of the results from three groups is less serious than a definite progression would have been, and not much can be expected of solutions from less than the whole material. Nevertheless the table suggests that correct weighting is of some importance.

For this purpose we might adopt the published observational probable errors and the cosmical P.E. corresponding to the published estimate of 0″·011 for cosmical $\bar{\tau}$. But the true probable errors may be greater than those published ; moreover, it seems advisable to use v-residuals as well as τ-components.

An increase in the observational probable errors is suggested by the material.* The τ-components and v-residuals for the bright stars are moderately well determined, and after rough allowance for observational error give a fair estimate of the true peculiar motion. Those for the faint stars are primarily observational errors, and after allowance for the cosmical scatter are larger than would be expected from the published probable errors.

For the short-period Cepheids the range in absolute magnitude is probably so small that reduction of the proper motions to a common apparent magnitude is nearly the same as their reduction to a common parallax. Thus the mean parallactic motion, $q \sin \lambda$, applies with satisfactory accuracy to each individual star, and we have at first sight an almost ideal case for the use of the v-residuals v' as well as τ-components. Unfortunately the former are considerably greater than the latter. Unweighted $\bar{\tau}$ equals 0″·0164, while the unweighted mean of the residuals from Mrs. Bok and Miss Boyd's parallactic solution is 0″·0198. The difference is still greater for the bright stars and very small for the faint ones. The discordant results obtained above for q from magnitude-groups discourage the use of v-residuals for these groups, and we give instead a table for precision-groups, explained in detail later, which give more accordant values of q. Group I contains the most exact reduced motions, mainly of bright stars, and Group III the least exact, mainly of faint stars. The grouping is much the same as that by magnitude, but better in principle and luckier in its effects. Table II gives for each group the number of stars, algebraic mean τ, signless mean τ, algebraic mean v' and signless mean v'. The last column is hardly changed if we use v-residuals from the parallactic solution given later in this paper.

The algebraic means are not intolerably large. The difference in Group I between the signless means $\bar{\tau}$ and $\overline{v'}$ is striking. It is, however,

TABLE II

Group	n	Alg. $\overline{\tau}$	$\bar{\tau}$	Alg. $\overline{v'}$	$\overline{v'}$
		″	″	″	″
I	20	−0·0041	0·0121	−0·0021	0·0181
II	18	−0·0061	0·0152	+0·0025	0·0196
III	20	+0·0098	0·0216	−0·0014	0·0218
All	58	+0·0001	0·0164	−0·0005	0·0198

* Mrs. Bok and Miss Boyd come to the same conclusion on other grounds.

not too large to be due to chance ; in this case the v-residuals should certainly be included. On the other hand, cosmical $\overline{v'}$ may really exceed cosmical $\bar{\tau}$. If this is so and the difference is due to star-streaming (which, however, can hardly explain the whole amount in Group I), then the v-residuals should again be included. (When the stars are uniformly distributed over the sky and a common parallax is deduced from reduced motions by comparing quadratic mean V' with a quadratic mean peculiar motion σ defined by $\sigma^2 = \frac{1}{2}(\overline{\tau^2} + \overline{v'^2}) = \frac{1}{2}\overline{\mu'^2}$, the effect of star-streaming is exactly eliminated, on the ellipsoidal theory, for any velocity-ellipsoid and any solar apex. The elimination is fairly good for moderate streaming when linear means are used.) The range in absolute magnitude is probably too small for the size of the v-residuals to be due to imperfect removal of parallactic motion. If there is a real difference due to some unknown cause, there is no *prima facie* reason why $\bar{\tau}$ should be preferred to $\overline{v'}$ as a measure of the peculiar proper motion. We therefore give the v-residuals the same weight as the τ-components.

The following is an outline of the method used. The published observational probable errors R_o, adjusted as described below, gave the probable errors $r_o (= 10^{0.2(m-11)} R_o)$ of the reduced motions. The stars were then grouped according to r_o. Group I contains 20 stars with r_o between 0″·0011 and 0″·0046 ; Group II, 18 with r_o between 0″·0051 and 0″·0082 ; Group III, 20 with r_o between 0″·0088 and 0″·0178. The value 0″·011 already mentioned for cosmical $\bar{\tau}$ corresponds with a cosmical P.E. $0{\cdot}845 \times 0''{\cdot}011 = 0''{\cdot}0093$. Thus cosmical scatter predominates in Groups I and II, observational in Group III ; this remains true when the increased observational and cosmical scatters evaluated in this paper are used. Suppose now that the published probable errors require multiplication by a factor k, and that the cosmical P.E. is r_c. Then the total P.E. r of a reduced motion in one co-ordinate is given by $r^2 = k^2 r_o^2 + r_c^2$. The values of k and r_c were determined by a simple method of approximation from the two conditions that

$$\frac{1}{2n} \Sigma \frac{|\tau| + |v'|}{r} = 1{\cdot}183$$

for the first and third groups separately (the average residual being 1·183 times the P.E. for a Gaussian distribution). It was then found that the relation was approximately satisfied for the second group, so that no slight adjustment of the values of k and r_c was necessary. The mean parallax from peculiar motions was found by using $\eta_c = 1{\cdot}183 r_c$ for the mean peculiar proper motion instead of $\bar{\tau}$ or $\overline{v'}$, and weights proportional to $1/r^2$ were assigned to the v's for the parallactic solution.

The published values of R_o have been used, except that all probable errors of 0″·006 and less have been counted as 0″·007.* This adjustment affects 12 stars ; in 11 cases the P.E. concerned refers to a meridian determination. It seems undesirable to allow the meridian determinations

* This refers to single determinations. Thus the minimum P.E. for means of two determinations is about 0″·005.

greater weight than the photographic ones. In a few cases where photographic and meridian motions have been combined the mean with the revised weights would differ from the published mean. We nevertheless use the published means, so as to try the effect of re-weighting the stars without actually changing their motions. In only two or three cases is there much difference between the meridian and photographic determinations. It has been assumed that the correction-factor k is the same for bright as for faint stars. This is, of course, an important assumption, but it seems inevitable and not unreasonable.

The values found from Groups I and III are $k = 1{\cdot}23$ and $r_c = 0''{\cdot}0119$ (*cf.* $0''{\cdot}0093$ mentioned above, from τ-components only). These values give an average residual (relative to P.E., τ and v' together) of 1·192 for Group II; this is sufficiently close to the theoretical 1·183, particularly as Group II contains the largest relative residual (4·9 times its P.E.). Thus the published probable errors require, on the basis of this solution, the increase mentioned in the preceding paragraph, and then a uniform increase of 23 per cent. The value $r_c = 0''{\cdot}0119$ corresponds with $\eta_c = 1{\cdot}183 r_c = 0''{\cdot}0141$. Using $\pi = 4{\cdot}74\eta_c/\overline{V'}$ with $\overline{V'} = 77$ km./sec., we find $\pi = 0''{\cdot}000868$, $M = +0^{m}{\cdot}69$. The high final value of M from peculiar motions is, of course, due to the inclusion of v-residuals as well as τ-components. Separate determinations have not been made, but it is evident that the result is a mean between something like $+0^{m}{\cdot}2$ from τ-components and $+1^{m}{\cdot}2$ from v-residuals. For this reason we refrain from attaching a P.E. to the mean value.

With the above values of k and r_c, the values of r lie between $0''{\cdot}012$ and $0''{\cdot}025$. Weights $w = (0''{\cdot}012/r)^2$ were assigned, the weights thus lying between 1 and 0·23. A solution for q by the formula $q = -\Sigma(wv \sin\lambda)/\Sigma(w \sin^2\lambda)$ gave $q = 0''{\cdot}0158$; then, using $\pi = 4{\cdot}74q/U$ with $U = 109$ km./sec., we have $\pi = 0''{\cdot}000688$, $M = +0^{m}{\cdot}19$. Results from the separate groups are given below.

TABLE III

Group	q	π	M
	″	″	m
I	0·0194	0·000842	+0·63
II	0·0089	0·000389	−1·05
III	0·0187	0·000813	+0·55
All	0·0158	0·000688	+0·19
B.B.	0·0148	0·000645	+0·05

The value of q differs little from, and generally confirms, that derived by Mrs. Bok and Miss Boyd. The present method leads, however, to a P.E. considerably greater than theirs ($0^{m}{\cdot}18$ in absolute magnitude), even allowing for our higher estimate of the observational and cosmical scatter. The present solution finds q with weight $\Sigma(w \sin^2\lambda) = 26{\cdot}13$. The P.E. of unit weight has been assumed to be $0''{\cdot}012$, as stated above, for the purpose of weighting; that derived from the v-residuals is, of course, somewhat greater, namely, $0''{\cdot}0136$. Thus the P.E. of q in our solution is $0''{\cdot}00235$ or $0''{\cdot}00266$. The corresponding P.E. in absolute magnitude is $0^{m}{\cdot}33$ or $0^{m}{\cdot}37$.

Perhaps we may take $0^m.35$ as approximately the P.E. in magnitude, excluding from consideration the uncertainty of U. The P.E. of the unweighted solution given in Table I is $0^m.38$, if derived from the residuals.

In view of this increased P.E. of π_{par} there is no apparent reason for supposing π_{pec} less reliable than π_{par}; in the author's opinion it is, if anything, more reliable. From the results, $M_{par} = +0^m.19$ and $M_{pec} = +0^m.69$, it is therefore tempting to conclude that Shapley's revised zero-point requires a positive correction of about $0^m.44$. But the P.E. of this result from all sources can hardly be much less than $0^m.3$, and it is evident that we are still far from knowing the scale of the universe with satisfactory precision. The purpose of this paper is not to propose any definite value for the zero-point correction, but to communicate the results yielded by one particular method of discussing the proper motions.

THE SUN'S ROTATION PERIOD DERIVED FROM RECURRENT SUNSPOTS, 1924–1933.

H. W. Newton.

(Communicated by the Astronomer Royal)

In *M.N.*, **85**, 548, 1925, is given a determination of the Sun's rotation period from measures made at Greenwich of long-lived sunspots observed during four 11-year solar cycles, 1878–1923. As similar data are now available for another cycle, 1924–1933, it is of interest to compare the derived rotation period for this cycle with the values already obtained, *loc. cit.*, for the four preceding cycles.

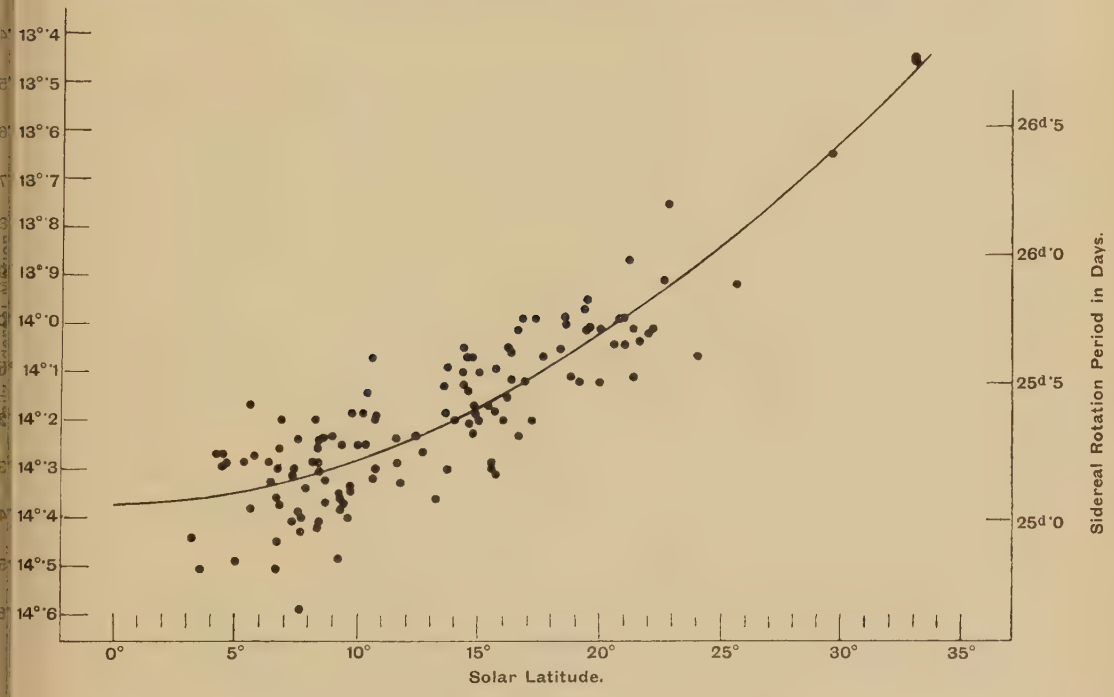
The material on which this determination is based is published in a convenient form in Ledger I of the annual *Greenwich Photoheliographic Results*; this section contains all groups of sunspots that last for a solar rotation or longer. In addition, individual components of groups are tabulated, and such spots which last to a second solar rotation provide the requisite data for this paper, as in the earlier one.

The longest lived components are almost invariably the leaders of groups; their stability of form, which is approximately circular, and their length of life make them very suitable for determining the Sun's rotation.

The method of reduction is as follows: The mean position of the spot for the days when visible at its first appearance (excluding days when it is more than 75° from the central meridian) is compared with the corresponding mean position after its reappearance. If the change of longitude be $x°$ in the interval of y days, then the angular motion of the spot is $x°/y$ per mean solar day, relative to a meridian on the Sun having a sidereal rotation of period 25·38 days (equivalent to a diurnal motion of 14°·18), this being the datum adopted in the *Nautical Almanac* for the computation of the longitude of the central meridian as seen from the Earth. Thus the sidereal angular motion of the spot per mean solar day is $14°.18 + x°/y$.

The cycle, 1924–1933, provides 130 spots suitable for this determination. They range in latitude from 3° to 33°, but there are few spots above 23°. With eight exceptions, the spots are necessarily the leading components of groups.* The results for the individual spots in the cycle are given in the diagram, in which the daily sidereal motions are plotted against latitudes.

As pointed out in previous investigations,† there is a considerable dispersion in the results as given by individual groups or individual spots.



The Sun's Rotation Period from Recurrent Sunspots, 1924–1933.

In the cases of individual spots, the dispersion arises in part from the non-uniformity of motion of the spot during its life-history—a motion that is related to the phase of development.‡ As data spread over a solar cycle include spots observed in all stages of development, no appreciable systematic error should arise from these characteristic motions.

Table I gives the results obtained for each 5° of latitude from the equator to 35°. The mean latitude (ϕ), the sidereal daily motion (ξ) and the number of spots (n) are given in each case. The value of the sidereal angular daily motion found, in the form, $a - b \sin^2 \phi$, from this data is

$$\xi = 14^\circ{\cdot}37 - 3^\circ{\cdot}00 \sin^2 \phi.$$

* The following component of a recurrent bipolar group lasts on the average for only 11 days, or one-quarter the length of life of the leader.

† *M.N.*, **65**, 825, 1905, and **85**, 550, 1925.

‡ *M.N.*, **85**, 553, 1925.

TABLE I

Mean Daily Sidereal Motions of Sunspots, 1924–1933

		N. Spots	S. Spots	Mean	O–C
0°–5°	ϕ	4°·7	3°·9	4°·2	
	ξ	14°·34	14°·38	14°·36	+0°·01
	n	3	4	7	
5°–10°	ϕ	7°·7	8°·1	7°·9	
	ξ	14°·31	14°·35	14°·33	+0°·02
	n	23	22	45	
10°–15°	ϕ	13°·3	12°·3	12°·7	
	ξ	14°·20	14°·19	14°·20	−0°·02
	n	14	16	30	
15°–20°	ϕ	16°·6	17°·7	17°·0	
	ξ	14°·10	14°·11	14°·10	−0°·01
	n	17	12	29	
20°–25°	ϕ	21°·9	21°·0	21°·5	
	ξ	13°·97	14°·04	14°·00	+0°·03
	n	8	7	15	
25°–30°	ϕ	29°·6	25°·6	27°·6	
	ξ	13°·59	13°·92	13°·75	+0°·02
	n	1	1	2	
30°–35°	ϕ	33°·0	...	33°·0	
	ξ	13°·45	...	13°·45	−0°·03
	n	2	...	2	

This value of ξ is in satisfactory agreement with the values derived respectively for the four previous cycles * as set forth in Table II.

TABLE II

Cycle	Date	Number of Sunspots	Diurnal Motion $= \xi$
I	1878–1888	89	$14^\circ{\cdot}36 - 2^\circ{\cdot}5 \sin^2 \phi$
II	1889–1899	122	$14^\circ{\cdot}39 - 3^\circ{\cdot}0 \sin^2 \phi$
III	1900–1913	99	$14^\circ{\cdot}39 - 2^\circ{\cdot}8 \sin^2 \phi$
IV	1914–1923	139	$14^\circ{\cdot}39 - 2^\circ{\cdot}6 \sin^2 \phi$
V	1924–1933	130	$14^\circ{\cdot}37 - 3^\circ{\cdot}0 \sin^2 \phi$

In view of the large secular change in the Sun's equatorial rotation period given by spectroscopic observations,† the consistent value of the *a* term derived from long-lived sunspots during five cycles is especially noteworthy.

Royal Observatory, Greenwich:
1934 *October* 26.

* The mean value of ξ for the epoch 1878–1923 similarly derived is $14^\circ{\cdot}37 - 2^\circ{\cdot}60 \sin^2 \phi$ (*Greenwich Photoheliographic Results*, 1924, and *M.N.*, **85**, 552, 1925).

† See *Trans. I.A.U.*, **4**, 43, 1933.

NOTE ON DAVIDSON'S METHOD OF DETERMINING THE TRUE ANOMALY AND PERIHELION-TIME IN HYPERBOLIC ORBITS.

C. H. Smiley, M.A., Ph.D.

It seems desirable to make a few comments on the Rev. M. Davidson's paper on the "Determination of the True Anomaly and the Time of Perihelion Passage in Hyperbolic Orbits." * Davidson obtains an approximation to the area of a sector described in hyperbolic motion by using four terms of a power series in $y=(1-e)/e$. His formulæ for the coefficients of the power series give no hint of the ease with which the coefficients of further terms of the series may be obtained. An easy method of obtaining as many coefficients as desired is given below. Furthermore, Davidson suggests that his method may be used for all hyperbolic orbits in which $e<1{\cdot}01$ and $v<158°$, but makes no statement as to the number of significant digits that can be obtained with certainty in the determination of the time-interval to perihelion by his method. The answer to this important question is also given below.

Davidson expands, by the binomial theorem, the expression on the right-hand side of the equation

$$r^2=\frac{p^2}{4e^2}\sec^4\frac{v}{2}\left(1+\frac{y}{2}\sec^2\frac{v}{2}\right)^{-2}.$$

Substituting this expansion in $\int(r^2/2)dv$, he obtains, by integrating from $v=0$ to $v=v$, an expression for the sector-area, namely,

$$\frac{p^2}{8e^2}\int_0^v\left(\sec^4\frac{v}{2}-2\left\{\frac{y}{2}\right\}\sec^6\frac{v}{2}+\ldots\binom{-2}{n}\left\{\frac{y}{2}\right\}^n\sec^{2n+4}\frac{v}{2}+\ldots\right)dv,$$

where $\binom{-2}{n}$ denotes the coefficient of x^n in the expansion of $(1+x)^{-2}$.

It may be pointed out that

$$\int\sec^{2n+2}\frac{v}{2}dv=\int\left(1+\tan^2\frac{v}{2}\right)^n\sec^2\frac{v}{2}dv$$

$$=2\left\{\tan\frac{v}{2}+\binom{n}{1}\tfrac{1}{3}\tan^3\frac{v}{2}+\ldots\binom{n}{k}\frac{1}{2k+1}\tan^{2k+1}\frac{v}{2}+\ldots\frac{1}{2n+1}\tan^{2n+1}\frac{v}{2}\right\}.$$

The limits of integration are, as previously indicated, $v=0$ to $v=v$.

* *M.N.*, **93**, 33, 1932.

In the notation adopted by Davidson the area of the sector is given by

$$\frac{p^2}{4e^2}S = \frac{p^2}{4e^2}(A - By + Cy^2 - Dy^3 + Ey^4 - \ . \ . \ .).$$

Then, if $\tan(v/2) = V$,

$$A = V + \frac{V^3}{3}, \qquad B = V + 2\frac{V^3}{3} + \frac{V^5}{5}, \qquad C = V + 3\frac{V^3}{3} + 3\frac{V^5}{5} + \frac{V^7}{7},$$

$$D = V + 4\frac{V^3}{3} + 6\frac{V^5}{5} + 4\frac{V^7}{7} + \frac{V^9}{9}, \qquad E = V + 5\frac{V^3}{3} + 10\frac{V^5}{5} + 10\frac{V^7}{7} + 5\frac{V^9}{9} + \frac{V^{11}}{11},$$

and, in general, the coefficient of y^n in the series S is given by the following expression :

$$\binom{-2}{n}\binom{1}{-2}^n \left\{ V + \binom{n+1}{1}\frac{V^3}{3} + \ . \ . \ . \binom{n+1}{k}\frac{V^{2k+1}}{2k+1} + \ . \ . \ . \frac{V^{2n+3}}{2n+3} \right\}.$$

Thus, to compute these coefficients, it is suggested that one first compute $V, V^3, V^5, V^7, \ . \ . \ . \ V^{2n+1}, \ . \ . \ .$, and then $V, \frac{V^3}{3}, \frac{V^5}{5}, \ . \ . \ . \ \frac{V^{2n+1}}{2n+1}$, etc. The coefficients of the power series in y are then easily obtained as linear combinations of the elements of the latter sequence.

As an illustrative example the coefficients A to E inclusive will be computed for the limiting case suggested by Davidson, namely, $e = 1{\cdot}01$ and $v = 158°$.

For this case $y = 0{\cdot}00990099$, and for the coefficients of the power series in y we shall need the odd powers of $\tan(v/2)$.

n	$\tan^n(v/2)$	$\frac{\tan^n(v/2)}{n}$
1	5·14455	5·14455
3	136·158	45·3860
5	3603·62	720·723
7	95374·9	13625·0
9	252423 × 10	280470
11	668075 × 10²	607341 × 10

and

$A =$ 50·5306	$A = 50{\cdot}5306$
$B =$ 816·640	$-By = 8{\cdot}0855$
$C =$ 11946·3	$+Cy^2 = 1{\cdot}1711$
$D = 169741$	$-Dy^3 = 0{\cdot}1647$
$E = 238108 \times 10$	$+Ey^4 = 0{\cdot}0229$
	$S = 59{\cdot}975$

From this example we may obtain information as to the precision to be obtained by Davidson's method at the limits named in his paper. Since, for hyperbolic eccentricities, all the terms of the series S are positive, the magni-

tude of the first term neglected in Davidson's method will serve as a measure of the accuracy obtained. It is seen that the term Ey^4 contributes two units to the fourth significant digit of S; hence, within the limits named, *only the third significant digit of* $t-T$ *obtained by Davidson's method can be relied upon.* In particular, if the perihelion distance q were one-tenth of an astronomical unit, the four terms of the power series used in Davidson's method yield, as the value of $t-T$, 154·00 days, which is in error by *over an hour.*

The series S ceases to be useful long before it ceases to converge. This is due to the rapidity with which the coefficients of the higher powers of y increase. It is suggested that the method of Gauss for nearly parabolic orbits, as amended by Marth, may easily be adapted to machine calculation. A table of the natural values of $1/B$, σ and $1/\nu^2$ * with argument A would be very useful.† One would have the following formulæ :—

$$M_i = M \cdot \frac{1}{B} \qquad \tan(v/2) = \gamma\sigma \tan(w/2),$$

where w is the parabolic anomaly corresponding to the final value of M. Then

$$\bar{x} = r\cos v = \frac{q}{\nu^2}\left(1 - \tan^2\frac{v}{2}\right),$$

$$\bar{y} = r\sin v = \frac{q}{\nu^2}\left(2\tan\frac{v}{2}\right).$$

It is seen from the discussion above that the four terms of the power series utilised by Davidson in his method are not sufficient for a high degree of accuracy in the determination of perihelion-times within the limits he named. Though, as has been shown, additional terms of the power series in y are easily obtained, it seems questionable whether the method offers any advantage over the classical Gauss-Marth method, provided tables suitable for machine calculation are available for the latter.

* Such a table is being constructed by Mr. Paul Herget of the Cincinnati Observatory ; at the present time it is nearing completion.

† Notation as in R. T. Crawford's *Determination of Orbits of Comets and Asteroids*, p. 29 (New York, 1930).

NOTE ON DR. SMILEY'S PAPER.

Rev. M. Davidson, B.A., D.Sc.

Dr. Smiley refers to my paper (*M.N.*, **93**, 33–39) in which y^4 and higher powers of y were neglected. These are very seldom required within the limit mentioned on p. 37—that is, 2·5 units from the Sun. When they are required they can be obtained with sufficient accuracy by extrapolation. It may be pointed out that Cy^2/By, Dy^3/Cy^2 and therefore Ey^4/Dy^3 are nearly equal and hence Ey^4 can be computed when it is required. Thus in the example taken by Smiley, which deals with a hypothetical case 3 units from the Sun, and therefore outside the limit which I laid down in my paper,

$$Cy^2/By = 1/6{\cdot}90, \qquad Dy^3/Cy^2 = 1/7{\cdot}11.$$

It may be assumed that $Ey^4/Dy^3 = 1/7{\cdot}32$ approximately and $Ey^4 = 0{\cdot}0225$, $Fy^5 = 0{\cdot}0003$. The value of Ey^4 differs by 4 units in the fourth decimal from the true value given by Smiley, and the error in such an exceptional case is less than 2 minutes. In dealing with actual cases where the eccentricity is always less than 1·01 the error is considerably smaller. Thus with Comet Ryves where $e = 1{\cdot}002065$ (*J.B.A.A.*, **44**, 68–73) if Smiley's figures for A, B, C, etc. are used

$$\begin{aligned} -By &= 1{\cdot}68310 \\ +Cy^2 &= \cdot05075 \\ -Dy^3 &= \cdot00149 \\ +Ey^4 &= \cdot00004 \end{aligned}$$

In this case $Cy^2/By = 1/33{\cdot}1$, $Dy^3/Cy^2 = 1/34{\cdot}1$, and the extrapolated value of Ey^4 is the same as Smiley would obtain by computing E. I have generally extrapolated when an error of more than 10 seconds occurred by neglecting y^4 and higher powers of y. In the example of Comet Reid on p. 36 of my previous paper the factor 1/50 could be used to obtain Ey^4, and the result is 6 units in the seventh decimal; this involves an error of 4 seconds only, so that the neglected term is insignificant.

The tables based on the Gauss-Marth method referred to by Smiley are in Bauschinger's *Tafeln*, xvii and xviii, Watson's Tables IX and X, etc.* Their adaptation for machine calculation, as suggested by Smiley, would be an advantage.† If Watson's Tables X are used it is generally necessary to recompute to find the accurate value of A. This applies in the determination of both the true anomaly and the time of perihelion passage, whereas in my tables the latter can be obtained directly without any approximate value as a start. Watson's Table IX is very convenient for finding the true anomaly directly; my tables can effect this only by successive approximations. The

* These tables are not strictly accurate, as they are based on a series in which the fourth and higher powers of $(1-e)/(1+e)$ are neglected.

† Table IX has natural numbers, but it is used in conjunction with Table VI which involves logarithms.

disadvantage about Watson's Table IX is that it cannot be used for values of V exceeding 3·7.

We may refer to Jens P. Möller's tables (*M.N.*, **93**, 777–788), which are also very convenient for finding the true anomaly. Unfortunately they are limited to $P = (t - T)q^{-3/2} = 1000$, and are not applicable to cases of small perihelion distances and large values of the true anomaly.

G. Stracke has recently compiled tables adapted for logarithms and machines. Tafeln 15^a–17 will be found useful for cases of high eccentricity.*

Errata

In my paper, **93**, 33 (1932), on p. 36, line 12 from the foot, *for* 0·093654 *read* 0·092654. As the value of A was only provisional the final result is correct.

In Table II in the last ten lines the expression D/C is given to 4 significant figures, though 3 will generally suffice. On recomputing these the following more accurate results are obtained :—

V	D/C
4·9	12·93
5·0	13·45
5·1	13·97

STELLAR PARALLAXES DETERMINED AT THE ROYAL OBSERVATORY, GREENWICH.

(Communicated by the Astronomer Royal)

The following list is a continuation of the results obtained with the 26-inch Thompson telescope, and published annually in the November number of the *Monthly Notices*.

Two values of the proper motion in R.A. are given. The first value is based on meridian observations, or the comparison of long-interval photographs. The second value is the relative proper motion derived from the parallax plates.

In the penultimate column the first value of the probable error refers to the parallax, the second refers to the relative proper motion.

In computing the probable error an average value of $\pm\,''\cdot027$ has been adopted as the probable error of a plate of unit weight. It is considered that this gives a truer indication of the real probable error than the somewhat fortuitous result derived from a comparatively small number of plates.

To minimize the effect of *quasi*-seasonal error the observations have been spread over not less than six epochs.

Generally six comparison stars of mean magnitude 11–12 have been

* Bauschinger, *Tafeln zur Theoretischen Astronomie*, Zweite Auflage, neubearbeitet von G. Stracke, 1934.

used, the magnitude of the parallax star being reduced to equality by means of a rotating sector and, for the brighter stars, a neutral-tinted filter absorbing 4·8 magnitudes in conjunction. Two exposures of 3 minutes each are made on each plate, the carrier being turned through 180° after the first exposure. As the filter is permanently mounted on one of the carriers, the reversal of the carrier should eliminate any distortion due to the filter.

For the brighter stars the visual magnitudes are from the *Draper Catalogue*, and for the fainter from other sources. The photographic magnitudes are from the *Greenwich Astrographic Catalogue*.

No.	Name and B.D. No.	Mag. Vis. Photog.	Spectral Type	R.A. Dec. 1900·0	Proper Motion R.A.	Proper Motion Dec.	Parallax	P.E.	No. of Plates
				h m s	″	″	″	″	
1	Br. 3217	6·2	A3	0 3 49	+·111	−·041	−·007	± ·009	22
	78° 1	6·1		+79° 10′	+·095			± ·005	
2	42 Cas	5·3	Ao	1 35 10	+·060	·000	+·001	± ·007	23
	69° 114	5·2		+70° 7′	+·089			± ·005	
3	48 Cas	4·6	A3	1 53 44	−·063	+·027	+·029	± ·006	31
	70° 153	4·8		+70° 25′	−·060			± ·003	
4	Rad. 606	8·4	Go	2 17 23	−·145	+·012	−·015	± ·006	33
	88° 11	8·7		+88° 15′	−·158			± ·002	
5	O.A. 3176	8·5	G5	2 45 38	+·152	−·108	+·023	± ·007	29
	79° 88	9·2		+79° 51′	+·156			± ·005	
6	A.C. 79° 1584	...		3 6 30	+·380	+·306	+·088	± ·007	24
		12·6		+79° 34′	+·416			± ·009	
7	Br. 431	5·5	Fo	3 7 37	+·085	−·089	+·010	± ·006	27
	77° 115	5·7		+77° 22′	+·058			± ·005	
8	A.C. 72° 1810	...		3 34 23	+·235	−·421	+·019	± ·007	27
		12·7		+72° 41′	+·200			± ·005	
9	P. IV 112	6·0	Fo	4 35 22	+·031	−·116	+·016	± ·006	30
	75° 189	6·2		+75° 46′	+·039			± ·004	
10	C. 678	8·5	Ko	4 47 56	−·006	−·018	−·002	± ·007	23
	84° 93	9·0		+85° 3′	+·009			± ·008	
11	L. 10580	6·9	Ko	5 36 23	·000	+·008	−·006	± ·006	31
	64° 544	8·0		+64° 43′	+·007			± ·006	
12	L. 10800	7·7	Ko	5 42 13	+·009	−·001	+·001	± ·006	28
	65° 499	8·4		+65° 8′	+·005			± ·006	
13	O.A. 6341	7·4	Ko	5 52 12	+·021	−·023	−·013	± ·006	27
	65° 509	8·1		+65° 3′	+·012			± ·006	
14	O.A. 6607–8	7·5	Ko	6 7 27	+·011	−·042	−·002	± ·006	28
	64° 575	8·3		+64° 52′	+·013			± ·005	
15	42 Cam	5·0	B3	6 40 32	+·008	+·005	+·007	± ·007	26
	67° 454	5·0		+67° 41′	−·010			± ·005	

No.	Name and B.D. No.	Mag. Vis. Photog.	Spectral Type	R.A. Dec. 1900·0	Proper Motion R.A.	Proper Motion Dec.	Parallax	P.E.	No. of Plates
				h m s	″	″	″	″	
16	*c* UMa	5·2	F8	9 6 26	−·001	−·032	+·033	± ·007	27
	62° 1058	5·4		+61° 50′	−·002			± ·003	
17	Gr. 1669	5·9	Ko	10 34 42	−·050	−·041	−·004	± ·008	25
	69° 583	6·9		+68° 58′	−·029			± ·004	
18	*α* UMa	1·9	Ko	10 57 34	−·117	−·074	+·024	± ·008	27
	62° 1161	3·0		+62° 17′	−·088			± ·003	
19	79° 371	10·3		11 37 40	−·228	−·028	−·004	± ·008	23
		10·0		+79° 34′	−·225			± ·004	
20	8 Dra	5·3	Fo	12 51 30	+·002	−·049	+·018	± ·008	34
	66° 778	5·4		+65° 59′	+·014			± ·003	
21	Gr. 2303	7·5	Ko	15 56 39	−·038	+·024	−·012	± ·008	38
	72° 703	8·0		+72° 41′	−·017			± ·003	
22	L. 29593	7·3	Ko	16 5 9	−·045	+·042	+·005	± ·008	37
	65° 1098	8·2		+64° 57′	−·020			± ·003	
23	L. 33393–7	6·4	Ko	17 57 45	−·011	+·040	−·023	± ·010	32
	75° 647	7·2		+75° 11′	+·001			± ·004	
24	Gr. 2930	7·1	Ko	19 37 16	+·020	+·019	−·021	± ·009	35
	74° 831	7·6		+74° 9′	+·013			± ·004	
25	L. 40523	7·2	Ko	20 41 36	+·033	+·044	+·003	± ·009	32
	81° 712	8·1		+81° 39′	+·034			± ·003	
26	L.F. 3736	7·2	F2	20 58 58	+·042	+·108	+·014	± ·008	30
	83° 596	7·4		+83° 33′	+·035			± ·005	
27	9 Cep	4·9	B2p	21 35 14	+·004	+·002	−·018	± ·008	29
	61° 2169	5·1		+61° 38′	−·007			± ·005	
28	L. 42796	8·9	Ko	21 48 17	+·117	+·139	+·045	± ·009	25
	74° 938	9·4		+74° 32′	+·099			± ·007	
29	16 Cep	5·1	F5	21 57 50	−·073	−·158	+·020	± ·009	26
	72° 1009	5·5		+72° 42′	−·068			± ·005	
30	L.F2. 334	8·5	Ko	23 52 55	−·119	−·025	+·010	± ·007	33
	80° 791	9·1		+80° 48′	−·146			± ·003	

STELLAR PARALLAXES DETERMINED PHOTOGRAPHICALLY AT THE ROYAL OBSERVATORY, CAPE OF GOOD HOPE. (SIXTH LIST.)

(Communicated by H.M. Astronomer)

Previous lists were published in *M.N.*, **89**, 497, 1929; **90**, 190, 1929; **91**, 822, 1931; **92**, 730, 1932; and **94**, 185, 1933.

For most of the stars the Durchmusterung number is given, B.D. down to $-23°$, Co.D. south of this limit to $-52°$, and C.P.D. south of $-52°$. The positions of the stars for 1900 are only approximate in many cases, being frequently derived from Durchmusterung positions without the application of proper-motion in reducing the position for 1875 or other epoch to 1900. There should, however, be no trouble in identifying the stars.

The magnitudes and spectral types, for stars contained in the Draper Catalogue, have been extracted from that catalogue. For the fainter stars they have been taken from other sources or have been estimated from the plates.

Two values of the proper-motion in R.A. are given. The first (as also the proper-motion in declination) is based on meridian observations or from the comparison of photographs taken at fairly distant epochs; the second value is the relative proper-motion derived from the parallax plates themselves. The stars on the observing programme have generally been selected on account of large proper-motion. It will be seen from the tabular data that for most of the stars the large proper-motion previously published has been confirmed, although in one or two cases this is not so. Four stars have been included because they were near and approximately equal in brightness to other stars on the observing list. These stars are indicated by the absence of meridian proper-motions. Sixteen stars have a negative relative parallax, including all the four stars just mentioned.

In the last column the first value of the probable error refers to the parallax, and the second to the proper-motion in R.A. These probable errors are based on the probable errors of a plate of unit weight given in the eighth column. The parallaxes are relative to the mean of 4 or 5 comparison stars which are generally between magnitude 11·0 and 12·0.

The list contains 7 parallaxes greater than $0''\cdot100$. Of these 4 appear to be new and refer to stars far south. The list includes the faint distant companion of α Centauri found by Innes.

No.	Name and D.M. No.	Mag. Vis. Phot.	Type	R.A. Dec. (1900)	Proper-Motion R.A. Unit ″·001	Proper-Motion Dec.	P.E. Unit Weight. Unit ″·001	Parallax. Unit ″·001	P.E. Unit ″·001
				h m s					
501	−24° 8	8·8	G5	0 2 24	+ 322	+ 85	20	+ 42	± 8
		9·3		−24° 22′·7	+ 332				± 8
502	−27° 37	7·85	G5	0 7 32	+ 288	+ 97	23	+ 15	± 10
		8·5		−27° 24′·8	+ 292				± 10
503	−67° 23	...	...	0 16 50	+ 269	+ 207	29	− 14	± 10
		10·3		−67° 2′·3	+ 223				± 6
504	−17° 63	9·0	...	0 23 34	− 388	− 220	25	+ 25	± 10
		...		−16° 46′·4	− 360				± 9
505	U.O.C. 48	...	...	0 50 55	0	+ 301	22	+ 59	± 8
		12·0		−52° 23′·0	+ 340				± 9
506	−51° 273	9·1	Ko	1 2 21	+ 485	− 10	19	+ 43	± 6
		9·5		−51° 31′·5	+ 469				± 7
507	−52° 291	10·8	...	1 23 24	+ 328	+ 57	19	+ 13	± 8
		11·5		−52° 33′·9	+ 316				± 8
508	−52° 305	10·0	...	1 28 9	+ 366	+ 250	13	+ 3	± 5
		11·7		−52° 25′·0	+ 392				± 5
509	−46° 473	9·7	Go	1 37 22	+ 230	− 30	22	+ 49	± 7
		9·9		−45° 55′·5	+ 244				± 8
510	−27° 605	9·1	G5	1 43 2	− 23	− 291	24	+ 51	± 8
		9·7		−27° 14′·5	− 131				± 9
511	χ Eridani	3·73	G5	1 52 4	+ 669	+ 285	21	+ 45	± 7
	−52° 394	4·51		−52° 6′·4	+ 676				± 7
512	U.O.C. 53	...	...	2 7 36	+ 387	− 400	17	+ 13	± 7
	Ci. 20, 152	12·0		−64° 58′·1	+ 15				± 7
513	−1° 306	8·8	...	2 9 30	+1005	− 80	24	0	± 8
		...		−1° 40′·0	+ 992				± 9
514	−34° 968	8·6	Go	2 35 37	+ 83	− 476	28	+ 14	± 10
		8·9		−34° 19′·2	+ 104				± 11
515	−53° 566	9·5	...	3 23 44	+ 321	0	25	+ 33	± 9
		11·9		−53° 31′·0	+ 249				± 10
516	−24° 1826	8·7	...	3 38 27	+ 46	− 373	20	+ 56	± 7
		...		−24° 46′·4	+ 67				± 7
517	−7° 699	8·3	...	3 49 42	0	+ 540	16	+ 65	± 6
		...		−7° 7′·9	+ 34				± 7

No.	Name and D.M. No.	Mag. Vis. Phot.	Type	R.A. Dec. (1900)	Proper-Motion R.A. Unit ″·001	Proper-Motion Dec. Unit ″·001	P.E. Unit Weight. Unit ″·001	Parallax. Unit ″·001	P.E. Unit ″·001
				h m s					
518	γ Eridani	3·19	K5	3 53 22	+ 67	− 112	31	+ 14	± 10
	−13° 781	4·37		−13° 47′·6	+ 62				± 11
519	ξ Eridani	5·23	A2	4 18 42	− 49	− 52	23	+ 14	± 8
	−4° 818	5·29		−3° 58′·6	− 33				± 8
520	−56° 767	6·9	Go	5 0 23	− 92	+ 720	23	+ 47	± 8
		8·2		−56° 14′·2	− 46				± 8
521	−59° 444	9·1	Go	5 11 32	+ 876	+ 540	25	+ 19	± 8
		9·6		−59° 46′·3	+ 939				± 10
522	−3° 1123	8·8	K2	5 26 23	+ 742	−2094	26	+146	± 10
		9·9		−3° 41′7	+ 849				± 9
523	U.O.C. 48	...	...	5 28 12	+ 62	+ 412	32	+ 63	± 11
	Ci. 20, 336	11·5		−54° 3′·6	+ 186				± 11
524	−72° 451	8·8	Ko	6 8 26	+ 36	− 410	27	+ 40	± 9
		9·8		−72° 29′·0	+ 2				± 10
525	A.J. 852–3, 54	...	...	6 59 34	− 118	− 812	25	+ 63	± 9
		11·0		−10° 20′·9	− 106				± 10
526	Boss 1875	Var.	Md	7 10 29	+ 100	+ 315	22	+ 2	± 7
	−44° 3227	3·4–6·2		−44° 28′·6	+ 109				± 7
527	A.J. 852–3, 58	...	...	7 16 42	− 29	+ 529	28	+ 43	± 10
		10·0		−12° 27′·5	− 140				± 11
528	−44° 3483	10·0	...	7 27 29	...	...	28	− 7	± 8
		...		−44° 12′·0	+ 18				± 9
529	−44° 3484	9·9	...	7 27 33	− 194	− 470	33	+ 12	± 10
		...		−44° 12′·0	− 154				± 11
530	−44° 3485	10·0	...	7 27 34	...	...	27	− 12	± 8
		...		−44° 10′·8	− 16				± 9
531	Ci. 20, 428	...	...	7 30 4	+ 325	− 540	28	+ 20	± 9
		10·0		−10° 10′·0	+ 437				± 10
532	−69° 757	8·3	Fo	7 38 24	− 290	− 540	30	+ 10	± 10
		8·6		−69° 50′·3	+ 6				± 11
533	−24° 6019	8·9	Go	7 44 41	+ 192	− 259	26	+ 27	± 9
		8·3		−24° 43′·0	+ 163				± 9
534	Ci. 20, 446	...	...	7 52 36	+ 237	− 350	27	+ 3	± 9
		10·0		−21° 25′·8	+ 224				± 9

No.	Name and D.M. No.	Mag. Vis. Phot.	Type	R.A. Dec. (1900)	Proper-Motion R.A. Unit ″·001	Proper-Motion Dec. Unit ″·001	P.E. Unit Weight. Unit ″·001	Parallax. Unit ″·001	P.E. Unit ″·001
				h m s					
535	−39° 3869	9·5	K5	7 56 27	+ 565	− 660	26	+ 71	± 8
		10·7		−39° 45′·0	+ 491				± 9
536	−41° 3810	10·3	G	8 5 44	− 425	+ 160	26	+ 11	± 8
		9·8		−41° 47′·0	− 374				± 10
537	−55° 1565	...	...	8 26 2	− 252	+ 225	29	− 21	± 10
		11·0		−55° 54′·2	− 181				± 11
538	−3° 2525	9·2	...	8 54 8	+ 449	− 580	27	+ 24	± 11
		...		−3° 37′·6	+ 330				± 9
539	−64° 997	9·5	...	9 3 10	− 102	+ 362	26	− 7	± 11
		10·2		−64° 57′·7	− 99				± 10
540	−58° 1695	9·1	Ko	9 51 23	− 379	+ 480	26	+ 41	± 8
		9·9		−58° 16′·0	− 407				± 8
541	−51° 4628	...	...	10 13 21	+ 637	0	27	− 27	± 9
		9·9		−51° 59′·5	+ 410				± 9
542	−62° 1580	9·4	...	10 23 28	− 243	− 370	22	+ 21	± 8
		...		−62° 22′·5	− 312				± 8
543	−55° 4033	8·9	Ko	10 55 18	− 544	− 130	32	+ 1	± 9
		9·4		−56° 5′·0	− 400				± 10
544	−17° 3326	8·1	Go	11 8 15	− 333	− 157	31	+ 6	± 12
		8·7		−17° 36′·9	− 354				± 14
545	−33° 7845	7·55	F5	11 31 42	− 297	+ 103	34	+ 3	± 11
		8·1		−34° 5′·8	− 186				± 14
546	−10° 3380	9·3	...	11 53 36	+ 130	− 286	22	− 18	± 8
		...		−11° 4′·5	+ 2				± 10
547	Boss 3137	5·63	G5	11 55 36	+ 109	− 479	20	+ 58	± 8
	−9° 3413	6·41		−9° 52′·6	+ 147				± 7
548	−24° 10236	7·48	Ko	12 9 34	− 308	− 34	24	+ 35	± 9
		8·4		−24° 13′·1	− 282				± 9
549	−26° 9470	8·2	Go	12 56 5	− 493	− 232	18	+ 5	± 6
		8·4		−26° 49′·8	− 466				± 4
550	−74° 1161	7·7	B9	14 9 44	+ 63	− 430	27	− 16	± 10
		7·7		−74° 42′·9	− 10				± 9
551	Prox. Centauri	...	...	14 22 57	−3653	+ 842	27	+751	± 9
		12		−62° 15′·0	−3732				± 11

No.	Name and D.M. No.	Mag. Vis. Phot.	Type	R.A. Dec. (1900)	Proper-Motion R.A. Unit ″·001	Proper-Motion Dec. Unit ″·001	P.E. Unit Weight. Unit ″·001	Parallax. Unit ″·001	P.E. Unit ″·001
				h m s					
552	−26° 10340	9·5	G5	14 25 42	+ 34	− 449	26	− 5	± 8
		9·5		−26° 8′·5	+ 173				± 7
553	−12° 4105	8·8	G5	14 32 45	+ 59	− 294	28	− 5	± 10
		9·6		−12° 38′·0	+ 46				± 9
554	μ Virginis	3·95	F5	14 37 47	+ 106	− 322	20	+ 73	± 8
	−5° 3936	4·37		−5° 13′·4	+ 125				± 8
555	−65° 2914	7·4	Go	14 45 43	− 323	− 28	28	+ 1	± 10
		8·0		−66° 0′·3	− 277				± 10
556	−4° 3783	6·00	F5	14 53 40	− 369	− 103	25	+ 12	± 11
		6·42		−4° 35′·2	− 334				± 11
557	−36° 9862	7·82	Go	14 57 0	− 36	− 405	28	+ 12	± 11
		8·2		−36° 31′·2	+ 21				± 10
558	Boss 3869	6·43	G5	15 7 38	− 393	− 79	20	+ 39	± 9
	−24° 11928	7·4		−24° 55′·9	− 379				± 8
559	U.O.C. 58, 28	...	...	15 14 0	− 507	− 71	22	+ 29	± 7
		12·5		−48° 27′·1	− 494				± 7
560	Anon.	...	...	15 14 16	...	...	28	− 17	± 9
		12·5		−48° 25′·5	− 24				± 9
561	Wolf 563	...	...	15 15 31	− 658	− 375	26	+ 17	± 8
	Ci. 20, 927	10·5		−12° 48′·7	− 600				± 10
562	−40° 9712	8·7	...	15 25 44	−1109	−1074	27	+161	± 8
		10·5		−40° 54′·4	−1163				± 10
563	Boss 3980	4·69	F5	15 34 19	− 185	− 266	17	+ 56	± 5
	−44° 10310	5·9		−44° 19′·8	− 156				± 6
564	β Triang. Aust.	3·04	Fo	15 46 20	− 201	− 388	22	+ 80	± 7
	−63° 3723	3·32		−63° 7′·3	− 167				± 7
565	−23° 12605	9·4	G5	15 52 40	− 274	− 139	22	+ 12	± 6
		8·9		−23° 37′·0	− 232				± 8
566	Boss 4067	5·53	F8	15 54 43	− 634	− 400	19	+ 28	± 5
	−16° 4196	6·03		−16° 14′·3	− 583				± 7
567	−31° 12603	8·0	Go	16 2 51	+ 13	− 310	26	+ 14	± 9
		7·6		−31° 49′·6	+ 6				± 11
568	ν Scorpii	4·29	B3	16 6 11	− 11	− 32	26	+ 18	± 8
	−19° 4333	4·12		−19° 12′·1	− 9				± 10

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				h m s					
569	−19° 4431	8·2	G5	16 42 42	− 109	− 343	22	+ 19	± 7
		8·9		−19° 6′·0	− 51				± 9
570	−46° 11288	9·7	G5	17 4 57	0	− 720	25	+ 19	± 7
		9·8		−46° 25′·2	− 50				± 8
571	U.O.C. 48	...	...	17 13 40	− 190	− 269	25	+ 35	± 7
		10·0		−52° 56′·0	− 189				± 9
572	Anon.	...	...	17 29 45	...	...	17	− 11	± 5
		11·0		−44° 14′·9	+ 16				± 6
573	−44° 11909	10·0	...	17 29 51	− 702	− 898	25	+199	± 8
		11·5		−44° 13′·4	− 706				± 9
574	U.O.C. 48	...	...	17 35 10	− 508	− 225	22	+ 24	± 6
	Ci. 20, 1055	11·5		−58° 4′·0	− 345				± 9
575	η Serpentis	3·42	Ko	18 16 8	− 554	− 700	30	+ 38	± 8
	−2° 4599	4·42		−2° 55′·5	− 504				± 11
576	−48° 12822	10·0	...	18 51 40	+ 157	− 483	23	+ 3	± 6
		11·8		−48° 23′·9	+ 31				± 8
577	Wolf 852	...	...	19 30 38	− 59	− 426	26	+ 15	± 8
	Ci. 20, 1160	10·5		−10° 14′·5	− 85				± 10
578	−12° 5594	9·1	K5	19 51 46	− 59	− 510	26	+ 30	± 7
		10·3		−12° 49′·2	− 89				± 11
579	Boss 5216	3·25	Go Ao	20 15 24	+ 35	+ 1	29	+ 5	± 8
	−15° 5629	3·81		−15° 5′·8	+ 74				± 11
580	−21° 5703	8·2	Go	20 17 42	+ 516	−1064	24	+ 26	± 8
		9·0		−21° 39′·7	+ 552				± 10
581	−10° 5549	9·3	...	20 51 18	− 59	− 330	19	− 23	± 6
		...		−10° 1′·9	− 66				± 6
582	α Octantis	5·24	F2	20 52 36	− 9	− 362	23	+ 12	± 9
	−77° 1474	6·5		−77° 24′·3	+ 67				± 9
583	−47° 13695	10·7	G5	21 0 52	− 212	− 550	21	+ 35	± 9
		10·6		−47° 42′·0	− 189				± 9
584	−60° 7528	10·0	...	21 52 17	+ 522	− 70	19	+ 94	± 7
		...		−60° 13′·0	+ 850				± 8
585	Boss 5770	5·39	G5	22 18 18	+ 134	− 342	17	+ 53	± 6
	−58° 7954	6·2		−58° 17′·7	+ 140				± 6

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				h m s					
586	−55° 9866	7·78	Go	22 29 46	+ 472	− 210	19	+ 29	± 7
		8·3		−55° 7′·3	+ 391				± 7
587	−32° 17191	7·49	Ko	22 35 15	+ 324	− 59	22	+ 57	± 8
		8·4		−32° 30′·8	+ 355				± 8
588	−47° 14307	6·26	Go	22 36 39	− 19	− 336	22	+ 35	± 7
		7·3		−47° 43′·5	− 21				± 7
589	−15° 6290	9·5	...	22 47 52	+ 972	− 560	26	+228	± 11
		...		−14° 47′·0	+1011				± 14
590	−83° 743	7·78	G5	22 52 30	+ 352	− 107	24	− 12	± 8
		8·6		−83° 14′·5	+ 339				± 7
591	Boss 5950	4·77	G5	23 1 18	+ 68	0	23	+ 32	± 9
	−24° 17497	5·55		−24° 17′·0	+ 59				± 8
592	−15° 6355	8·5	F5	23 3 23	+ 71	− 315	16	− 3	± 6
		8·9		−15° 35′·2	+ 103				± 5
593	−45° 14980	9·7	K2	23 10 39	+ 455	+ 160	24	+ 22	± 10
		10·2		−45° 5′·9	+ 502				± 9
594	−14° 6448	5·27	G5	23 13 51	+ 295	− 89	27	+ 27	± 9
	(S. comp.)	6·05		−14° 0′·2	+ 245				± 7
595	−11° 6064	8·1	Ko	23 17 49	+ 462	+ 248	19	+ 39	± 8
		9·1		−11° 19′·2	+ 458				± 5
596	−47° 14628	8·5	G5	23 30 9	+ 193	− 330	24	+ 42	± 9
		9·4		−47° 29′·4	+ 118				± 6
597	−73° 2299	6·7	Ko	23 33 42	+ 117	− 750	17	+133	± 7
		7·7		−73° 15′·3	+ 94				± 5
598	−33° 16646	7·16	Ko	23 34 34	+ 120	− 343	29	+ 38	± 13
		7·7		−33° 17′·4	+ 105				± 12
599	Boss 6153	5·07	Ko	23 53 33	− 57	− 67	22	− 6	± 9
	−4° 5996	6·07		−4° 6′·6	− 56				± 8
600	−37° 15492	8·3	K5	23 59 40	+5637	−2359	37	+219	± 14
		10·1		−37° 51′·6	+5501				± 14